Maple 2018.2 Integration Test Results
on the problems in "1 Algebraic functions/1.1 Binomial products/1.1.2 Quadratic"
Test results for the 277 problems in "1.1.2.2 (c $x)^{\wedge} m\left(a+b x^{\wedge} 2\right)^{\wedge} p . t x t^{\prime \prime}$

Problem 21: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(b x^{2}+a\right)^{5}}{x^{13}} \mathrm{~d} x
$$

Optimal (type 1, 17 leaves, 1 step):

$$
-\frac{\left(b x^{2}+a\right)^{6}}{12 a x^{12}}
$$

Result (type 1, 57 leaves):

$$
-\frac{b^{5}}{2 x^{2}}-\frac{5 a b^{4}}{4 x^{4}}-\frac{5 a^{3} b^{2}}{4 x^{8}}-\frac{a^{4} b}{2 x^{10}}-\frac{a^{5}}{12 x^{12}}-\frac{5 a^{2} b^{3}}{3 x^{6}}
$$

Problem 26: Result more than twice size of optimal antiderivative.

$$
\int x\left(b x^{2}+a\right)^{8} \mathrm{~d} x
$$

Optimal (type 1,14 leaves, 1 step):

$$
\frac{\left(b x^{2}+a\right)^{9}}{18 b}
$$

Result(type 1, 90 leaves):

$$
\frac{1}{18} b^{8} x^{18}+\frac{1}{2} a b^{7} x^{16}+2 a^{2} b^{6} x^{14}+\frac{14}{3} a^{3} b^{5} x^{12}+7 a^{4} b^{4} x^{10}+7 a^{5} b^{3} x^{8}+\frac{14}{3} a^{6} b^{2} x^{6}+2 a^{7} b x^{4}+\frac{1}{2} a^{8} x^{2}
$$

Problem 29: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(b x^{2}+a\right)^{8}}{x^{19}} \mathrm{~d} x
$$

Optimal(type 1, 17 leaves, 1 step):

$$
-\frac{\left(b x^{2}+a\right)^{9}}{18 a x^{18}}
$$

Result(type 1, 90 leaves):

$$
-\frac{b^{8}}{2 x^{2}}-\frac{2 a^{6} b^{2}}{x^{14}}-\frac{2 a b^{7}}{x^{4}}-\frac{7 a^{3} b^{5}}{x^{8}}-\frac{a^{7} b}{2 x^{16}}-\frac{a^{8}}{18 x^{18}}-\frac{7 a^{4} b^{4}}{x^{10}}-\frac{14 a^{5} b^{3}}{3 x^{12}}-\frac{14 a^{2} b^{6}}{3 x^{6}}
$$

Problem 30: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(b x^{2}+a\right)^{8}}{x^{21}} \mathrm{~d} x
$$

Optimal(type 1, 36 leaves, 3 steps):

$$
-\frac{\left(b x^{2}+a\right)^{9}}{20 a x^{20}}+\frac{b\left(b x^{2}+a\right)^{9}}{180 a^{2} x^{18}}
$$

Result(type 1, 90 leaves):

$$
-\frac{4 a^{5} b^{3}}{x^{14}}-\frac{b^{8}}{4 x^{4}}-\frac{7 a^{2} b^{6}}{2 x^{8}}-\frac{7 a^{6} b^{2}}{4 x^{16}}-\frac{4 a^{7} b}{9 x^{18}}-\frac{28 a^{3} b^{5}}{5 x^{10}}-\frac{35 a^{4} b^{4}}{6 x^{12}}-\frac{4 a b^{7}}{3 x^{6}}-\frac{a^{8}}{20 x^{20}}
$$

Problem 51: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{15}}{\left(b x^{2}+a\right)^{10}} \mathrm{~d} x
$$

Optimal(type 1, 35 leaves, 3 steps):

$$
\frac{x^{16}}{18 a\left(b x^{2}+a\right)^{9}}+\frac{x^{16}}{144 a^{2}\left(b x^{2}+a\right)^{8}}
$$

Result(type 1, 132 leaves):

$$
\begin{aligned}
& -\frac{21 a^{2}}{8 b^{8}\left(b x^{2}+a\right)^{4}}+\frac{7 a}{6 b^{8}\left(b x^{2}+a\right)^{3}}-\frac{7 a^{6}}{16 b^{8}\left(b x^{2}+a\right)^{8}}-\frac{1}{4 b^{8}\left(b x^{2}+a\right)^{2}}+\frac{3 a^{5}}{2 b^{8}\left(b x^{2}+a\right)^{7}}-\frac{35 a^{4}}{12 b^{8}\left(b x^{2}+a\right)^{6}}+\frac{7 a^{3}}{2 b^{8}\left(b x^{2}+a\right)^{5}} \\
& \quad+\frac{a^{7}}{18 b^{8}\left(b x^{2}+a\right)^{9}}
\end{aligned}
$$

Problem 90: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{x}}{-x^{2}+1} \mathrm{~d} x
$$

Optimal(type 3, 11 leaves, 4 steps):

$$
-\arctan (\sqrt{x})+\operatorname{arctanh}(\sqrt{x})
$$

Result(type 3, 23 leaves):

$$
-\frac{\ln (\sqrt{x}-1)}{2}+\frac{\ln (\sqrt{x}+1)}{2}-\arctan (\sqrt{x})
$$

Problem 91: Result more than twice size of optimal antiderivative.

$$
\int x^{m}\left(b x^{2}+a\right)^{5} \mathrm{~d} x
$$

Optimal(type 3, 97 leaves, 2 steps):

$$
\frac{a^{5} x^{1+m}}{1+m}+\frac{5 a^{4} b x^{3+m}}{3+m}+\frac{10 a^{3} b^{2} x^{5+m}}{5+m}+\frac{10 a^{2} b^{3} x^{7+m}}{7+m}+\frac{5 a b^{4} x^{9+m}}{9+m}+\frac{b^{5} x^{11+m}}{11+m}
$$

Result(type 3, 431 leaves):
$\frac{1}{(11+m)(9+m)(7+m)(5+m)(3+m)(1+m)}\left(x^{1+m}\left(b^{5} m^{5} x^{10}+25 b^{5} m^{4} x^{10}+5 a b^{4} m^{5} x^{8}+230 b^{5} m^{3} x^{10}+135 a b^{4} m^{4} x^{8}+950 b^{5} m^{2} x^{10}\right.\right.$ $+10 a^{2} b^{3} m^{5} x^{6}+1310 a b^{4} m^{3} x^{8}+1689 b^{5} m x^{10}+290 a^{2} b^{3} m^{4} x^{6}+5610 a b^{4} m^{2} x^{8}+945 b^{5} x^{10}+10 a^{3} b^{2} m^{5} x^{4}+3020 a^{2} b^{3} m^{3} x^{6}+10205 a b^{4} m x^{8}$ $+310 a^{3} b^{2} m^{4} x^{4}+13660 a^{2} b^{3} m^{2} x^{6}+5775 a b^{4} x^{8}+5 a^{4} b m^{5} x^{2}+3500 a^{3} b^{2} m^{3} x^{4}+25770 a^{2} b^{3} m x^{6}+165 a^{4} b m^{4} x^{2}+17300 a^{3} b^{2} m^{2} x^{4}+14850 a^{2} b^{3} x^{6}$ $+a^{5} m^{5}+2030 a^{4} b m^{3} x^{2}+34890 a^{3} b^{2} m x^{4}+35 a^{5} m^{4}+11310 a^{4} b m^{2} x^{2}+20790 a^{3} b^{2} x^{4}+470 a^{5} m^{3}+26765 a^{4} b m x^{2}+3010 a^{5} m^{2}+17325 a^{4} b x^{2}$ $\left.+9129 a^{5} m+10395 a^{5}\right)$ )

Problem 93: Unable to integrate problem.

$$
\int \frac{x^{m}}{\left(b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Optimal(type 5, 37 leaves, 1 step):

$$
\frac{x^{1+m} \text { hypergeom }\left(\left[2, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{a^{2}(1+m)}
$$

Result(type 8, 15 leaves):

$$
\int \frac{x^{m}}{\left(b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Problem 94: Unable to integrate problem.

$$
\int \frac{(c x)^{1+m}}{b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 5, 42 leaves, 1 step):

$$
\frac{(c x)^{2+m} \text { hypergeom }\left(\left[1,1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{a c(2+m)}
$$

Result(type 8, 19 leaves):

$$
\int \frac{(c x)^{1+m}}{b x^{2}+a} \mathrm{~d} x
$$

Problem 95: Unable to integrate problem.

$$
\int \frac{(c x)^{m}}{b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 5, 42 leaves, 1 step):

$$
\frac{(c x)^{1+m} \text { hypergeom }\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{a c(1+m)}
$$

Result(type 8, 17 leaves):

$$
\int \frac{(c x)^{m}}{b x^{2}+a} \mathrm{~d} x
$$

Problem 96: Unable to integrate problem.

$$
\int \frac{(c x)^{-1+m}}{b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 5, 36 leaves, 1 step):

$$
\frac{(c x)^{m} \text { hypergeom }\left(\left[1, \frac{m}{2}\right],\left[1+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{a c m}
$$

Result(type 8, 19 leaves):

$$
\int \frac{(c x)^{-1+m}}{b x^{2}+a} \mathrm{~d} x
$$

Problem 112: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(b x^{2}+a\right)^{9 / 2}}{x^{11}} \mathrm{~d} x
$$

Optimal(type 3, 103 leaves, 8 steps):

$$
-\frac{21 b^{3}\left(b x^{2}+a\right)^{3 / 2}}{128 x^{4}}-\frac{21 b^{2}\left(b x^{2}+a\right)^{5 / 2}}{160 x^{6}}-\frac{9 b\left(b x^{2}+a\right)^{7 / 2}}{80 x^{8}}-\frac{\left(b x^{2}+a\right)^{9 / 2}}{10 x^{10}}-\frac{63 b^{5} \operatorname{arctanh}\left(\frac{\sqrt{b x^{2}+a}}{\sqrt{a}}\right)}{256 \sqrt{a}}-\frac{63 b^{4} \sqrt{b x^{2}+a}}{256 x^{2}}
$$

Result(type 3, 212 leaves):

$$
\begin{aligned}
& -\frac{\left(b x^{2}+a\right)^{11 / 2}}{10 a x^{10}}-\frac{b\left(b x^{2}+a\right)^{11 / 2}}{80 a^{2} x^{8}}-\frac{b^{2}\left(b x^{2}+a\right)^{11 / 2}}{160 a^{3} x^{6}}-\frac{b^{3}\left(b x^{2}+a\right)^{11 / 2}}{128 a^{4} x^{4}}-\frac{7 b^{4}\left(b x^{2}+a\right)^{11 / 2}}{256 a^{5} x^{2}}+\frac{7 b^{5}\left(b x^{2}+a\right)^{9 / 2}}{256 a^{5}}+\frac{9 b^{5}\left(b x^{2}+a\right)^{7 / 2}}{256 a^{4}} \\
& \quad+\frac{63 b^{5}\left(b x^{2}+a\right)^{5 / 2}}{1280 a^{3}}+\frac{21 b^{5}\left(b x^{2}+a\right)^{3 / 2}}{256 a^{2}}-\frac{63 b^{5} \ln \left(\frac{2 a+2 \sqrt{a} \sqrt{b x^{2}+a}}{x}\right)}{256 \sqrt{a}}+\frac{63 b^{5} \sqrt{b x^{2}+a}}{256 a}
\end{aligned}
$$

Problem 163: Result more than twice size of optimal antiderivative.

$$
\int \sqrt{c x} \sqrt{-2 a x^{2}+3 a} \mathrm{~d} x
$$

Optimal(type 4, 77 leaves, 5 steps):

$$
-\frac{66^{1 / 4} a \text { EllipticE }\left(\frac{\sqrt{3-x \sqrt{6}} \sqrt{6}}{6}, \sqrt{2}\right) \sqrt{c x} \sqrt{-2 x^{2}+3}}{5 \sqrt{x} \sqrt{-2 a x^{2}+3 a}}+\frac{2(c x)^{3 / 2} \sqrt{-2 a x^{2}+3 a}}{5 c}
$$

Result(type 4, 228 leaves):

$$
\begin{aligned}
& \frac{1}{10 x\left(2 x^{2}-3\right)}\left(\sqrt { c x } \sqrt { - a ( 2 x ^ { 2 } - 3 ) } \left(2 \sqrt{(-2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3} \sqrt{3} \sqrt{-x \sqrt{2} \sqrt{3}} \sqrt{(2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3} \sqrt{2} \operatorname{EllipticE}\left(\frac{1}{6}(\sqrt{3} \sqrt{2}\right.}} \begin{array}{l}
\left.\sqrt{(2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3}}), \frac{\sqrt{2}}{2}\right)-\sqrt{(-2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3}} \sqrt{3} \sqrt{-x \sqrt{2} \sqrt{3}} \operatorname{EllipticF}\left(\frac{\sqrt{3} \sqrt{2} \sqrt{(2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3}}}{6}\right. \\
\left.\left.\left.\frac{\sqrt{2}}{2}\right) \sqrt{(2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3}} \sqrt{2}+8 x^{4}-12 x^{2}\right)\right)
\end{array}, l\right.\right.
\end{aligned}
$$

Problem 172: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{c x}}{\sqrt{-2 a x^{2}+3 a}} d x
$$

Optimal(type 4, 53 leaves, 4 steps):

$$
-\frac{6^{1 / 4} \text { EllipticE }\left(\frac{\sqrt{3-x \sqrt{6}} \sqrt{6}}{6}, \sqrt{2}\right) \sqrt{c x} \sqrt{-2 x^{2}+3}}{\sqrt{x} \sqrt{-2 a x^{2}+3 a}}
$$

Result(type 4, 164 leaves):

$$
\begin{aligned}
& \frac{1}{12 a x\left(2 x^{2}-3\right)}\left(\sqrt{c x} \sqrt{-a\left(2 x^{2}-3\right)} \sqrt{2} \sqrt{(2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3} \sqrt{(-2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3}} \sqrt{3} \sqrt{-x \sqrt{2} \sqrt{3}}\left(2 \text { EllipticE } \left(\frac{1}{6}(\sqrt{3} \sqrt{2}\right.\right.}\right. \\
& \left.\left.\left.\quad \sqrt{(2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3}}), \frac{\sqrt{2}}{2}\right)-\operatorname{EllipticF}\left(\frac{\sqrt{3} \sqrt{2} \sqrt{(2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3}}}{6}, \frac{\sqrt{2}}{2}\right)\right)\right)
\end{aligned}
$$

Problem 173: Result more than twice size of optimal antiderivative.

$$
\int \frac{(c x)^{5 / 2}}{\left(-2 a x^{2}+3 a\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 86 leaves, 5 steps):

$$
\frac{c(c x)^{3 / 2}}{2 a \sqrt{-2 a x^{2}+3 a}}+\frac{33^{1 / 4} c^{2} \text { EllipticE }\left(\frac{\sqrt{3-x \sqrt{6}} \sqrt{6}}{6}, \sqrt{2}\right) \sqrt{c x} \sqrt{-2 x^{2}+3} 2^{1 / 4}}{4 a \sqrt{x} \sqrt{-2 a x^{2}+3 a}}
$$

Result(type 4, 229 leaves):

$$
\begin{aligned}
& -\frac{1}{16 x a^{2}\left(2 x^{2}-3\right)}\left(c ^ { 2 } \sqrt { c x } \sqrt { - a ( 2 x ^ { 2 } - 3 ) } \left(2 \sqrt{(-2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3} \sqrt{3} \sqrt{-x \sqrt{2} \sqrt{3}} \sqrt{(2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3} \sqrt{2} \text { EllipticE }\left(\frac{1}{6}(\sqrt{3} \sqrt{2}\right.}} \begin{array}{l}
\left.\sqrt{(2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3}}), \frac{\sqrt{2}}{2}\right)-\sqrt{(-2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3}} \sqrt{3} \sqrt{-x \sqrt{2} \sqrt{3}} \text { EllipticF }\left(\frac{\sqrt{3} \sqrt{2} \sqrt{(2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3}}}{6}\right. \\
\left.\left.\left.\frac{\sqrt{2}}{2}\right) \sqrt{(2 x+\sqrt{2} \sqrt{3}) \sqrt{2} \sqrt{3}} \sqrt{2}+8 x^{2}\right)\right)
\end{array} . \quad .\right.\right.
\end{aligned}
$$

Problem 174: Unable to integrate problem.

$$
\int x^{m}\left(b x^{2}+a\right)^{3 / 2} \mathrm{~d} x
$$

Optimal(type 5, 46 leaves, 2 steps):

$$
\frac{x^{1+m}\left(b x^{2}+a\right)^{5 / 2} \text { hypergeom }\left(\left[1,3+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{a(1+m)}
$$

Result(type 8, 15 leaves):

$$
\int x^{m}\left(b x^{2}+a\right)^{3 / 2} \mathrm{~d} x
$$

Problem 175: Unable to integrate problem.

$$
\int \frac{x^{2+m}}{\sqrt{b x^{2}+a}} d x
$$

Optimal(type 5, 46 leaves, 2 steps):

$$
\frac{x^{3+m} \text { hypergeom }\left(\left[1,2+\frac{m}{2}\right],\left[\frac{5}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right) \sqrt{b x^{2}+a}}{a(3+m)}
$$

Result(type 8, 17 leaves):

$$
\int \frac{x^{2+m}}{\sqrt{b x^{2}+a}} \mathrm{~d} x
$$

Problem 176: Unable to integrate problem.

$$
\int \frac{x^{1+m}}{\sqrt{b x^{2}+a}} \mathrm{~d} x
$$

Optimal(type 5, 46 leaves, 2 steps):

$$
\frac{x^{2+m} \text { hypergeom }\left(\left[1, \frac{3}{2}+\frac{m}{2}\right],\left[2+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right) \sqrt{b x^{2}+a}}{a(2+m)}
$$

Result(type 8, 17 leaves):

$$
\int \frac{x^{1+m}}{\sqrt{b x^{2}+a}} \mathrm{~d} x
$$

Problem 177: Unable to integrate problem.

$$
\int\left(\frac{a(2+m) x^{1+m}}{\sqrt{b x^{2}+a}}+\frac{b(3+m) x^{3+m}}{\sqrt{b x^{2}+a}}\right) \mathrm{d} x
$$

Optimal(type 3, 15 leaves, ? steps):

$$
x^{2+m} \sqrt{b x^{2}+a}
$$

Result(type 8, 41 leaves):

$$
\int\left(\frac{a(2+m) x^{1+m}}{\sqrt{b x^{2}+a}}+\frac{b(3+m) x^{3+m}}{\sqrt{b x^{2}+a}}\right) \mathrm{d} x
$$

Problem 179: Unable to integrate problem.

$$
\int\left(-\frac{b x^{1+m}}{\left(b x^{2}+a\right)^{3 / 2}}+\frac{m x^{-1+m}}{\sqrt{b x^{2}+a}}\right) \mathrm{d} x
$$

Optimal (type 3, 13 leaves, ? steps):

$$
\frac{x^{m}}{\sqrt{b x^{2}+a}}
$$

Result (type 8, 36 leaves):

$$
\int\left(-\frac{b x^{1+m}}{\left(b x^{2}+a\right)^{3 / 2}}+\frac{m x^{-1+m}}{\sqrt{b x^{2}+a}}\right) \mathrm{d} x
$$

Problem 181: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{1 / 3}}{x^{3}} \mathrm{~d} x
$$

Optimal(type 3, 78 leaves, 6 steps):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 3}}{2 x^{2}}-\frac{b \ln (x)}{6 a^{2 / 3}}+\frac{b \ln \left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{4 a^{2 / 3}}-\frac{b \arctan \left(\frac{\left(a^{1 / 3}+2\left(b x^{2}+a\right)^{1 / 3}\right) \sqrt{3}}{3 a^{1 / 3}}\right) \sqrt{3}}{6 a^{2 / 3}}
$$

Result(type 8, 55 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 3}}{2 x^{2}}+\frac{\left(\int \frac{b}{3 x\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 182: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{1 / 3}}{x^{5}} \mathrm{~d} x
$$

Optimal(type 3, 102 leaves, 7 steps):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 3}}{4 x^{4}}-\frac{b\left(b x^{2}+a\right)^{1 / 3}}{12 a x^{2}}+\frac{b^{2} \ln (x)}{18 a^{5 / 3}}-\frac{b^{2} \ln \left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{12 a^{5 / 3}}+\frac{b^{2} \arctan \left(\frac{\left(a^{1 / 3}+2\left(b x^{2}+a\right)^{1 / 3}\right) \sqrt{3}}{3 a^{1 / 3}}\right) \sqrt{3}}{18 a^{5 / 3}}
$$

Result(type 8, 72 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 3}\left(b x^{2}+3 a\right)}{12 x^{4} a}+\frac{\left(\int-\frac{b^{2}}{9 a x\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 183: Unable to integrate problem.

$$
\int x^{2}\left(b x^{2}+a\right)^{1 / 3} \mathrm{~d} x
$$

Optimal(type 4, 229 leaves, 4 steps):

$$
\begin{aligned}
& \frac{6 a x\left(b x^{2}+a\right)^{1 / 3}}{55 b}+\frac{3 x^{3}\left(b x^{2}+a\right)^{1 / 3}}{11}+\frac{1}{55 b^{2} x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(6 3 ^ { 3 / 4 } a ^ { 2 } \left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 /}\right.\right.} \\
& \left.\quad \text { 3 EllipticF }\left(\frac{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}+\left(b x^{2}+a\right)^{2 / 3}}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\right)
\end{aligned}
$$

Result(type 8, 68 leaves):

$$
\frac{3 x\left(5 b x^{2}+2 a\right)\left(b x^{2}+a\right)^{1 / 3}}{55 b}+\frac{\left(\int-\frac{6 a^{2}}{55 b\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 185: Unable to integrate problem.

$$
\int\left(b x^{2}+a\right)^{2 / 3} \mathrm{~d} x
$$

Optimal(type 4, 430 leaves, 5 steps):
$\frac{3 x\left(b x^{2}+a\right)^{2 / 3}}{7}-\frac{12 a x}{7\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)}$

$\left.-\mathrm{I} \sqrt{3}) \sqrt{2} \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}+\left(b x^{2}+a\right)^{2 / 3}}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\right)+\frac{1}{7 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(63^{1 / 4} a^{4 / 3}\left(a^{1 / 3}\right.\right.$
$\left.-\left(b x^{2}+a\right)^{1 / 3}\right)$ EllipticE $\left.\left(\frac{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}+\left(b x^{2}+a\right)^{2 / 3}}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\right)$
Result(type 8, 27 leaves):

$$
\frac{3 x\left(b x^{2}+a\right)^{2 / 3}}{7}+\int \frac{4 a}{7\left(b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 187: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{4 / 3}}{x} \mathrm{~d} x
$$

Optimal(type 3, 84 leaves, 7 steps):

$$
\frac{3 a\left(b x^{2}+a\right)^{1 / 3}}{2}+\frac{3\left(b x^{2}+a\right)^{4 / 3}}{8}-\frac{a^{4 / 3} \ln (x)}{2}+\frac{3 a^{4 / 3} \ln \left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{4}-\frac{a^{4 / 3} \arctan \left(\frac{\left(a^{1 / 3}+2\left(b x^{2}+a\right)^{1 / 3}\right) \sqrt{3}}{3 a^{1 / 3}}\right) \sqrt{3}}{2}
$$

Result(type 8, 15 leaves):

$$
\int \frac{\left(b x^{2}+a\right)^{4 / 3}}{x} \mathrm{~d} x
$$

Problem 188: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{4 / 3}}{x^{3}} \mathrm{~d} x
$$

Optimal(type 3, 89 leaves, 7 steps):

$$
2 b\left(b x^{2}+a\right)^{1 / 3}-\frac{\left(b x^{2}+a\right)^{4 / 3}}{2 x^{2}}-\frac{2 a^{1 / 3} b \ln (x)}{3}+a^{1 / 3} b \ln \left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)-\frac{2 a^{1 / 3} b \arctan \left(\frac{\left(a^{1 / 3}+2\left(b x^{2}+a\right)^{1 / 3}\right) \sqrt{3}}{3 a^{1 / 3}}\right) \sqrt{3}}{3}
$$

Result(type 8, 66 leaves):

$$
-\frac{a\left(b x^{2}+a\right)^{1 / 3}}{2 x^{2}}+\frac{\left(\int \frac{b\left(3 b x^{2}+4 a\right)}{3 x\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 189: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{4 / 3}}{x^{5}} \mathrm{~d} x
$$

Optimal(type 3, 99 leaves, 7 steps):

$$
-\frac{b\left(b x^{2}+a\right)^{1 / 3}}{3 x^{2}}-\frac{\left(b x^{2}+a\right)^{4 / 3}}{4 x^{4}}-\frac{b^{2} \ln (x)}{9 a^{2 / 3}}+\frac{b^{2} \ln \left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{6 a^{2 / 3}}-\frac{b^{2} \arctan \left(\frac{\left(a^{1 / 3}+2\left(b x^{2}+a\right)^{1 / 3}\right) \sqrt{3}}{3 a^{1 / 3}}\right) \sqrt{3}}{9 a^{2 / 3}}
$$

Result(type 8, 67 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 3}\left(7 b x^{2}+3 a\right)}{12 x^{4}}+\frac{\left(\int \frac{2 b^{2}}{9 x\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 190: Unable to integrate problem.

$$
\int\left(b x^{2}+a\right)^{4 / 3} \mathrm{~d} x
$$

Optimal(type 4, 224 leaves, 4 steps):

$$
\frac{24 a x\left(b x^{2}+a\right)^{1 / 3}}{55}+\frac{3 x\left(b x^{2}+a\right)^{4 / 3}}{11}-\frac{1}{55 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(1 6 3 ^ { 3 / 4 } a ^ { 2 } \left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 /}\right.\right.
$$

$\left.{ }^{3}\right)$ EllipticF $\left.\left(\frac{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}+\left(b x^{2}+a\right)^{2 / 3}}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\right)$
Result(type 8, 62 leaves):

$$
\frac{3 x\left(5 b x^{2}+13 a\right)\left(b x^{2}+a\right)^{1 / 3}}{55}+\frac{\left(\int \frac{16 a^{2}}{55\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 191: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{4 / 3}}{x^{4}} \mathrm{~d} x
$$

Optimal(type 4, 223 leaves, 4 steps):

$$
-\frac{8 b\left(b x^{2}+a\right)^{1 / 3}}{9 x}-\frac{\left(b x^{2}+a\right)^{4 / 3}}{3 x^{3}}-\frac{1}{27 x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(1 6 b \left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 /}\right.\right.
$$

$\left.\left.{ }^{3}\right) \operatorname{EllipticF}\left(\frac{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}+\left(b x^{2}+a\right)^{2 / 3}}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) 3^{3 / 4}\right)$
Result(type 8, 64 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 3}\left(11 b x^{2}+3 a\right)}{9 x^{3}}+\frac{\left(\int \frac{16 b^{2}}{27\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 194: Unable to integrate problem.

$$
\int \frac{1}{x^{3}\left(b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Optimal(type 3, 81 leaves, 6 steps):

$$
-\frac{\left(b x^{2}+a\right)^{2 / 3}}{2 a x^{2}}+\frac{b \ln (x)}{6 a^{4 / 3}}-\frac{b \ln \left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{4 a^{4 / 3}}-\frac{b \arctan \left(\frac{\left(a^{1 / 3}+2\left(b x^{2}+a\right)^{1 / 3}\right) \sqrt{3}}{3 a^{1 / 3}}\right) \sqrt{3}}{6 a^{4 / 3}}
$$

Result(type 8, 38 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{2 / 3}}{2 a x^{2}}+\int-\frac{b}{3 a x\left(b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 195: Unable to integrate problem.

$$
\int \frac{x^{4}}{\left(b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Optimal(type 4, 456 leaves, 6 steps):
$-\frac{27 a x\left(b x^{2}+a\right)^{2 / 3}}{91 b^{2}}+\frac{3 x^{3}\left(b x^{2}+a\right)^{2 / 3}}{13 b}-\frac{81 a^{2} x}{91 b^{2}\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)}$
$-\frac{1}{91 b^{3} x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(273^{3 / 4} a^{7 / 3}\left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)\right.$ EllipticF $\left(\frac{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}\right.$
$\left.-\mathrm{I} \sqrt{3}) \sqrt{2} \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}+\left(b x^{2}+a\right)^{2 / 3}}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\right)+\frac{1}{182 b^{3} x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(813^{1 / 4} a^{7 / 3}\left(a^{1 / 3}\right.\right.$
$\left.-\left(b x^{2}+a\right)^{1 / 3}\right)$ EllipticE $\left.\left.\left(\frac{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}+\left(b x^{2}+a\right)^{2 / 3}}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}\left(\frac{\sqrt{6}}{2}\right.}+\frac{\sqrt{2}}{2}\right)\right)$
Result(type 8, 45 leaves):

$$
-\frac{3 x\left(-7 b x^{2}+9 a\right)\left(b x^{2}+a\right)^{2 / 3}}{91 b^{2}}+\int \frac{27 a^{2}}{91 b^{2}\left(b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 196: Unable to integrate problem.

$$
\int \frac{1}{\left(b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Optimal(type 4, 417 leaves, 4 steps):
$-\frac{3 x}{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}$

$$
\begin{aligned}
& -\frac{1}{b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(3 ^ { 3 / 4 } a ^ { 1 / 3 } ( a ^ { 1 / 3 } - ( b x ^ { 2 } + a ) ^ { 1 / 3 } ) \operatorname { E l l i p t i c F } \left(\frac{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}\right.\right. \\
& \left.-\mathrm{I} \sqrt{3}) \sqrt{2} \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}+\left(b x^{2}+a\right)^{2 / 3}}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\right)+\frac{1}{2 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(3 3 ^ { 1 / 4 } a ^ { 1 / 3 } \left(a^{1 / 3}\right.\right. \\
& \left.\left.-\left(b x^{2}+a\right)^{1 / 3}\right) \text { EllipticE }\left(\frac{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}+\left(b x^{2}+a\right)^{2 / 3}}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\right)
\end{aligned}
$$

Result(type 8, 11 leaves):

$$
\int \frac{1}{\left(b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 198: Unable to integrate problem.

$$
\int \frac{x^{4}}{\left(b x^{2}+a\right)^{2 / 3}} \mathrm{~d} x
$$

Optimal(type 4, 232 leaves, 4 steps):
$-\frac{27 a x\left(b x^{2}+a\right)^{1 / 3}}{55 b^{2}}+\frac{3 x^{3}\left(b x^{2}+a\right)^{1 / 3}}{11 b}-\frac{1}{55 b^{3} x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(273^{3 / 4} a^{2}\left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 /}\right.\right.$
$\left.\left.{ }^{3}\right) \operatorname{EllipticF}\left(\frac{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}+\left(b x^{2}+a\right)^{2 / 3}}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\right)$
Result(type 8, 68 leaves):

$$
-\frac{3 x\left(-5 b x^{2}+9 a\right)\left(b x^{2}+a\right)^{1 / 3}}{55 b^{2}}+\frac{\left(\int \frac{27 a^{2}}{55 b^{2}\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 199: Unable to integrate problem.

$$
\int \frac{1}{x^{2}\left(b x^{2}+a\right)^{2 / 3}} \mathrm{~d} x
$$

Optimal(type 4, 213 leaves, 3 steps):

$$
\begin{aligned}
& -\frac{\left(b x^{2}+a\right)^{1 / 3}}{a x}+\frac{1}{3 a x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(( a ^ { 1 / 3 } - ( b x ^ { 2 } + a ) ^ { 1 / 3 } ) \operatorname { E l l i p t i c F } \left(\frac{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}\right.\right. \\
& \quad-\mathrm{I} \sqrt{3}) \sqrt{\left.\frac{a^{2 / 3}+a^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}+\left(b x^{2}+a\right)^{2 / 3}}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) 3^{3 / 4}\right)}
\end{aligned}
$$

Result(type 8, 58 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 3}}{a x}+\frac{\left(\int-\frac{b}{3 a\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 200: Unable to integrate problem.

$$
\int \frac{1}{x^{4}\left(b x^{2}+a\right)^{2 / 3}} d x
$$

Optimal(type 4, 232 leaves, 4 steps):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 3}}{3 a x^{3}}+\frac{7 b\left(b x^{2}+a\right)^{1 / 3}}{9 a^{2} x}-\frac{1}{27 a^{2} x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(7 b \left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 /}\right.\right.
$$

$\left.{ }^{3}\right)$ EllipticF $\left.\left(\frac{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}+\left(b x^{2}+a\right)^{2 / 3}}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) 3^{3 / 4}\right)$
Result(type 8, 70 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 3}\left(-7 b x^{2}+3 a\right)}{9 a^{2} x^{3}}+\frac{\left(\int \frac{7 b^{2}}{27 a^{2}\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 201: Unable to integrate problem.

$$
\int \frac{1}{x\left(b x^{2}+a\right)^{4 / 3}} \mathrm{~d} x
$$

Optimal(type 3, 75 leaves, 6 steps):

$$
\frac{3}{2 a\left(b x^{2}+a\right)^{1 / 3}}-\frac{\ln (x)}{2 a^{4 / 3}}+\frac{3 \ln \left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{4 a^{4 / 3}}+\frac{\arctan \left(\frac{\left(a^{1 / 3}+2\left(b x^{2}+a\right)^{1 / 3}\right) \sqrt{3}}{3 a^{1 / 3}}\right) \sqrt{3}}{2 a^{4 / 3}}
$$

Result(type 8, 15 leaves):

$$
\int \frac{1}{x\left(b x^{2}+a\right)^{4 / 3}} \mathrm{~d} x
$$

Problem 202: Unable to integrate problem.

$$
\int \frac{1}{x^{4}\left(b x^{2}+a\right)^{4 / 3}} d x
$$

Optimal(type 4, 471 leaves, 7 steps):

$$
\begin{aligned}
& \frac{3}{2 a x^{3}\left(b x^{2}+a\right)^{1 / 3}}-\frac{11\left(b x^{2}+a\right)^{2 / 3}}{6 a^{2} x^{3}}+\frac{55 b\left(b x^{2}+a\right)^{2 / 3}}{18 a^{3} x}+\frac{55 b^{2} x}{18 a^{3}\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)} \\
& +\frac{1}{54 a^{8 / 3} x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(5 5 b ( a ^ { 1 / 3 } - ( b x ^ { 2 } + a ) ^ { 1 / 3 } ) \operatorname { E l l i p t i c F } \left(\frac{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}\right.\right. \\
& \left.-\mathrm{I} \sqrt{3}) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}+\left(b x^{2}+a\right)^{2 / 3}}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}} 3^{3 / 4} \sqrt{2}\right)-\frac{1}{36 a^{8 / 3} x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(5 5 b \left(a^{1 / 3}\right.\right. \\
& \left.-\left(b x^{2}+a\right)^{1 / 3}\right) \operatorname{EllipticE}\left(\frac{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}+\left(b x^{2}+a\right)^{2 / 3}}{\left(-\left(b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)
\end{aligned}
$$

Result(type 8, 66 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{2 / 3}\left(-14 b x^{2}+3 a\right)}{9 a^{3} x^{3}}+\int-\frac{b\left(14 b x^{2}-13 a\right)}{27 a^{3}\left(x^{2}+\frac{a}{b}\right)\left(b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 203: Unable to integrate problem.

$$
\int(c x)^{1 / 3}\left(b x^{2}+a\right)^{1 / 3} \mathrm{~d} x
$$

Optimal(type 3, 96 leaves, 4 steps):

$$
\frac{(c x)^{4 / 3}\left(b x^{2}+a\right)^{1 / 3}}{2 c}-\frac{a c^{1 / 3} \ln \left(b^{1 / 3}(c x)^{2 / 3}-c^{2 / 3}\left(b x^{2}+a\right)^{1 / 3}\right)}{4 b^{2 / 3}}-\frac{a c^{1 / 3} \arctan \left(\frac{\left(1+\frac{2 b^{1 / 3}(c x)^{2 / 3}}{c^{2 / 3}\left(b x^{2}+a\right)^{1 / 3}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{6 b^{2 / 3}}
$$

Result(type 8, 78 leaves):

$$
\frac{x\left(b x^{2}+a\right)^{1 / 3}(c x)^{1 / 3}}{2}+\frac{\left(\int \frac{a x}{3\left(c^{2} x^{2}\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right)(c x)^{1 / 3}\left(c^{2} x^{2}\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{x\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 206: Unable to integrate problem.

$$
\int(c x)^{7 / 3}\left(b x^{2}+a\right)^{4 / 3} \mathrm{~d} x
$$

Optimal(type 3, 143 leaves, 6 steps):

$$
\frac{a^{2} c(c x)^{4 / 3}\left(b x^{2}+a\right)^{1 / 3}}{27 b}+\frac{a(c x)^{10 / 3}\left(b x^{2}+a\right)^{1 / 3}}{9 c}+\frac{(c x)^{10 / 3}\left(b x^{2}+a\right)^{4 / 3}}{6 c}+\frac{a^{3} c^{7 / 3} \ln \left(b^{1 / 3}(c x)^{2 / 3}-c^{2 / 3}\left(b x^{2}+a\right)^{1 / 3}\right)}{2 b^{5 / 3}}
$$

$$
+\frac{2 a^{3} c^{7 / 3} \arctan \left(\frac{\left(1+\frac{2 b^{1 / 3}(c x)^{2 / 3}}{c^{2 / 3}\left(b x^{2}+a\right)^{1 / 3}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{81 b^{5 / 3}}
$$

Result(type 8, 113 leaves):

$$
\frac{x\left(9 b^{2} x^{4}+15 a b x^{2}+2 a^{2}\right)\left(b x^{2}+a\right)^{1 / 3} c^{2}(c x)^{1 / 3}}{54 b}+\frac{\left(\int-\frac{4 a^{3} x}{81 b\left(c^{2} x^{2}\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right) c^{2}(c x)^{1 / 3}\left(c^{2} x^{2}\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{x\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 207: Unable to integrate problem.

$$
\int(c x)^{1 / 3}\left(b x^{2}+a\right)^{4 / 3} \mathrm{~d} x
$$

Optimal(type 3, 120 leaves, 5 steps):

$$
\begin{aligned}
& \frac{a(c x)^{4 / 3}\left(b x^{2}+a\right)^{1 / 3}}{3 c}+\frac{(c x)^{4 / 3}\left(b x^{2}+a\right)^{4 / 3}}{4 c}-\frac{a^{2} c^{1 / 3} \ln \left(b^{1 / 3}(c x)^{2 / 3}-c^{2 / 3}\left(b x^{2}+a\right)^{1 / 3}\right)}{6 b^{2 / 3}} \\
& \quad-\frac{a^{2} c^{1 / 3} \arctan \left(\frac{\left(1+\frac{2 b^{1 / 3}(c x)^{2 / 3}}{c^{2 / 3}\left(b x^{2}+a\right)^{1 / 3}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{9 b^{2 / 3}}
\end{aligned}
$$

Result(type 8, 90 leaves):

$$
\frac{x\left(3 b x^{2}+7 a\right)\left(b x^{2}+a\right)^{1 / 3}(c x)^{1 / 3}}{12}+\frac{\left(\int \frac{2 a^{2} x}{9\left(c^{2} x^{2}\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right)(c x)^{1 / 3}\left(c^{2} x^{2}\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{x\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 208: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{4 / 3}}{(c x)^{11 / 3}} \mathrm{~d} x
$$

Optimal(type 3, 114 leaves, 5 steps):

$$
-\frac{3 b\left(b x^{2}+a\right)^{1 / 3}}{2 c^{3}(c x)^{2 / 3}}-\frac{3\left(b x^{2}+a\right)^{4 / 3}}{8 c(c x)^{8 / 3}}-\frac{3 b^{4 / 3} \ln \left(b^{1 / 3}(c x)^{2 / 3}-c^{2 / 3}\left(b x^{2}+a\right)^{1 / 3}\right)}{4 c^{11 / 3}}-\frac{b^{4 / 3} \arctan \left(\frac{\left(1+\frac{2 b^{1 / 3}(c x)^{2 / 3}}{c^{2 / 3}\left(b x^{2}+a\right)^{1 / 3}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{2 c^{11 / 3}}
$$

Result(type 8, 92 leaves):

$$
-\frac{3\left(b x^{2}+a\right)^{1 / 3}\left(5 b x^{2}+a\right)}{8 x^{2} c^{3}(c x)^{2 / 3}}+\frac{\left(\int \frac{b^{2} x}{\left(c^{2} x^{2}\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right)\left(c^{2} x^{2}\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{c^{3}(c x)^{2 / 3}\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 210: Unable to integrate problem.

$$
\int(c x)^{4 / 3}\left(b x^{2}+a\right)^{4 / 3} \mathrm{~d} x
$$

Optimal(type 4, 469 leaves, 6 steps):

$$
\begin{aligned}
& \frac{16 a^{2} c(c x)^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}}{135 b}+\frac{8 a(c x)^{7 / 3}\left(b x^{2}+a\right)^{1 / 3}}{45 c}+\frac{(c x)^{7 / 3}\left(b x^{2}+a\right)^{4 / 3}}{5 c}-\left(8 a ^ { 2 } c ^ { 1 / 3 } ( c x ) ^ { 1 / 3 } ( b x ^ { 2 } + a ) ^ { 1 / 3 } \left(c^{2 / 3}\right.\right. \\
& \left.-\frac{b^{1 / 3}(c x)^{2 / 3}}{\left(b x^{2}+a\right)^{1 / 3}}\right) / \sqrt{\frac{\left(c^{2 / 3}-\frac{b^{1 / 3}(c x)^{2 / 3}(1-\sqrt{3})}{\left(c^{2 / 3}-\frac{b^{1 / 3}(c x)^{2 / 3}(1+\sqrt{3})}{\left(b x^{2}+a\right)^{1 / 3}}\right)^{2}}\right.}{\left(c^{2 / 3}\right.}} c^{2 / 3}
\end{aligned}
$$

$$
\begin{aligned}
& \left.-\frac{b^{1 / 3}(c x)^{2 / 3}(1+\sqrt{3})}{\left(b x^{2}+a\right)^{1 / 3}}\right) \text { EllipticF } \sqrt{1-\frac{\left(c^{2 / 3}-\frac{b^{1 / 3}(c x)^{2 / 3}(1-\sqrt{3})}{\left(b x^{2}+a\right)^{1 / 3}}\right)^{2}}{\left(c^{2 / 3}-\frac{b^{1 / 3}(c x)^{2 / 3}(1+\sqrt{3})}{\left(b x^{2}+a\right)^{1 / 3}}\right)^{2}}, \frac{\sqrt{6}}{4}} \\
& +\frac{\sqrt{2}}{4}\left(\sqrt{\left.\left.\frac{c^{4 / 3}+\frac{b^{2 / 3}(c x)^{4 / 3}}{\left(b x^{2}+a\right)^{2 / 3}+\frac{b^{1 / 3} c^{2 / 3}(c x)^{2 / 3}}{\left(b x^{2}+a\right)^{1 / 3}}}}{\left(c^{2 / 3}-\frac{b^{1 / 3}(c x)^{2 / 3}(1+\sqrt{3})}{1^{1 / 3}}\right)^{2}} 3^{3 / 4}\right) /\left(b x^{2}+a\right)^{1^{1 / 3}}\right) / 405\left(c^{2 / 3}\right.}{ }^{\left.\left.-\frac{b^{1 / 3}(c x)^{2 / 3}\left(c^{2 / 3}-\frac{b^{1 / 3}(c x)^{2 / 3}}{\left(b x^{2}+a\right)^{1 / 3}}\right)}{\left(b x^{2}+a\right)^{1 / 3}}\right) b \sqrt{-\frac{b^{1 / 3}(c x)^{2 / 3}(1-\sqrt{3})}{\left(b x^{2}+a\right)^{1 / 3}\left(c^{2 / 3}-\frac{b^{1 / 3}(c x)^{2 / 3}(1+\sqrt{3})}{\left(b x^{2}+a\right)^{1 / 3}}\right)^{2}}}\right)}\right.
\end{aligned}
$$

Result(type 8, 107 leaves):

$$
\frac{\left(27 b^{2} x^{4}+51 a b x^{2}+16 a^{2}\right)\left(b x^{2}+a\right)^{1 / 3} c(c x)^{1 / 3}}{135 b}+\frac{\left(\int-\frac{16 a^{3}}{405 b\left(c^{2} x^{2}\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right) c(c x)^{1 / 3}\left(c^{2} x^{2}\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{x\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 211: Unable to integrate problem.

$$
\int(c x)^{2 / 3}\left(b x^{2}+a\right)^{4 / 3} \mathrm{~d} x
$$

Optimal(type 5, 47 leaves, 2 steps):

$$
\frac{3 a(c x)^{5 / 3}\left(b x^{2}+a\right)^{1 / 3} \text { hypergeom }\left(\left[-\frac{4}{3}, \frac{5}{6}\right],\left[\frac{11}{6}\right],-\frac{b x^{2}}{a}\right)}{5 c\left(1+\frac{b x^{2}}{a}\right)^{1 / 3}}
$$

Result(type 8, 83 leaves):

$$
\frac{3 x^{2}\left(7 b x^{2}+15 a\right)\left(b x^{2}+a\right)^{1 / 3} c}{91(c x)^{1 / 3}}+\frac{\left(\int \frac{16 a^{2} x}{91\left(c x\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right) c\left(c x\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{\left(b x^{2}+a\right)^{2 / 3}(c x)^{1 / 3}}
$$

Problem 212: Unable to integrate problem.

$$
\int \frac{(c x)^{13 / 3}}{\left(b x^{2}+a\right)^{2 / 3}} \mathrm{~d} x
$$

Optimal(type 3, 124 leaves, 5 steps):

$$
-\frac{5 a c^{3}(c x)^{4 / 3}\left(b x^{2}+a\right)^{1 / 3}}{12 b^{2}}+\frac{c(c x)^{10 / 3}\left(b x^{2}+a\right)^{1 / 3}}{4 b}-\frac{5 a^{2} c^{13 / 3} \ln \left(b^{1 / 3}(c x)^{2 / 3}-c^{2 / 3}\left(b x^{2}+a\right)^{1 / 3}\right)}{12 b^{8 / 3}}
$$

$$
-\frac{5 a^{2} c^{13 / 3} \arctan \left(\frac{\left(1+\frac{2 b^{1 / 3}(c x)^{2 / 3}}{c^{2 / 3}\left(b x^{2}+a\right)^{1 / 3}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{18 b^{8 / 3}}
$$

Result(type 8, 102 leaves):

$$
-\frac{x\left(-3 b x^{2}+5 a\right)\left(b x^{2}+a\right)^{1 / 3} c^{4}(c x)^{1 / 3}}{12 b^{2}}+\frac{\left(\int \frac{5 a^{2} x}{9 b^{2}\left(c^{2} x^{2}\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right) c^{4}(c x)^{1 / 3}\left(c^{2} x^{2}\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{x\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 215: Unable to integrate problem.

$$
\int \frac{1}{(c x)^{8 / 3}\left(b x^{2}+a\right)^{2 / 3}} \mathrm{~d} x
$$

Optimal(type 4, 427 leaves, 4 steps):

$$
\begin{aligned}
& -\frac{3\left(b x^{2}+a\right)^{1 / 3}}{5 a c(c x)^{5 / 3}}-\left(33^{3 / 4} b(c x)^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}\left(c^{2 / 3}-\frac{b^{1 / 3}(c x)^{2 / 3}}{\left(b x^{2}+a\right)^{1 / 3}}\right) \sqrt{\frac{\left(c^{2 / 3}-\frac{b^{1 / 3}(c x)^{2 / 3}(1-\sqrt{3})}{\left(b x^{2}+a\right)^{1 / 3}}\right)^{2}}{\left(c^{2 / 3}-\frac{b^{1 / 3}(c x)^{2 / 3}(1+\sqrt{3})}{\left(b x^{2}+a\right)^{1 / 3}}\right.}\left(c^{2 / 3}\right.}\right. \\
& \left.-\frac{b^{1 / 3}(c x)^{2 / 3}(1+\sqrt{3})}{\left(b x^{2}+a\right)^{1 / 3}}\right) \text { EllipticF } \sqrt{1-\frac{\left(c^{2 / 3}-\frac{b^{1 / 3}(c x)^{2 / 3}(1-\sqrt{3})}{\left(b x^{2}+a\right)^{1 / 3}}\right)^{2}}{\left(c^{2 / 3}-\frac{b^{1 / 3}(c x)^{2 / 3}(1+\sqrt{3})}{\left(b x^{2}+a\right)^{1 / 3}}\right)^{2}}, \frac{\sqrt{6}}{4}}
\end{aligned}
$$

$$
\left.\left.-\frac{b^{1 / 3}(c x)^{2 / 3}(1-\sqrt{3})}{\left(b x^{2}+a\right)^{1 / 3}}\right) a^{2} c^{11 / 3} \sqrt{-\frac{b^{1 / 3}(c x)^{2 / 3}\left(c^{2 / 3}-\frac{b^{1 / 3}(c x)^{2 / 3}}{\left(b x^{2}+a\right)^{1 / 3}}\right)}{\left(b x^{2}+a\right)^{1 / 3}\left(c^{2 / 3}-\frac{b^{1 / 3}(c x)^{2 / 3}(1+\sqrt{3})}{\left(b x^{2}+a\right)^{1 / 3}}\right)^{2}}}\right)
$$

Result(type 8, 88 leaves):

$$
-\frac{3\left(b x^{2}+a\right)^{1 / 3}}{5 a x c^{2}(c x)^{2 / 3}}+\frac{\left(\int-\frac{3 b}{5 a\left(c^{2} x^{2}\left(b x^{2}+a\right)^{2}\right)^{1 / 3}} \mathrm{~d} x\right)\left(c^{2} x^{2}\left(b x^{2}+a\right)^{2}\right)^{1 / 3}}{c^{2}(c x)^{2 / 3}\left(b x^{2}+a\right)^{2 / 3}}
$$

Problem 216: Unable to integrate problem.

$$
\int x^{4}\left(b x^{2}+a\right)^{1 / 4} \mathrm{~d} x
$$

Optimal(type 4, 126 leaves, 5 steps):
$-\frac{4 a^{2} x\left(b x^{2}+a\right)^{1 / 4}}{77 b^{2}}+\frac{2 a x^{3}\left(b x^{2}+a\right)^{1 / 4}}{77 b}+\frac{2 x^{5}\left(b x^{2}+a\right)^{1 / 4}}{11}$

$$
+\frac{8 a^{7 / 2}\left(1+\frac{b x^{2}}{a}\right)^{3 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}{77 \cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) b^{5 / 2}\left(b x^{2}+a\right)^{3 / 4}}
$$

Result(type 8, 79 leaves):

$$
-\frac{2 x\left(-7 b^{2} x^{4}-a b x^{2}+2 a^{2}\right)\left(b x^{2}+a\right)^{1 / 4}}{77 b^{2}}+\frac{\left(\int \frac{4 a^{3}}{\left.77 b^{2}\left(\left(b x^{2}+a\right)^{3}\right)^{1 / 4} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{3}\right)^{1 / 4}}\right.}{\left(b x^{2}+a\right)^{3 / 4}}
$$

Problem 217: Unable to integrate problem.

$$
\int \frac{\left(-b x^{2}+a\right)^{1 / 4}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 4, 93 leaves, 3 steps):

$$
-\frac{\left(-b x^{2}+a\right)^{1 / 4}}{x}-\frac{\left(1-\frac{b x^{2}}{a}\right)^{3 / 4} \sqrt{\cos \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{a} \sqrt{b}}{\cos \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)\left(-b x^{2}+a\right)^{3 / 4}}
$$

Result(type 8, 86 leaves):

$$
-\frac{\left(-b x^{2}+a\right)^{1 / 4}\left(\left(-b x^{2}+a\right)^{3}\right)^{1 / 4}}{x\left(-\left(b x^{2}-a\right)^{3}\right)^{1 / 4}}+\frac{\left(\int-\frac{b}{2\left(-\left(b x^{2}-a\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(-b x^{2}+a\right)^{3}\right)^{1 / 4}}{\left(-b x^{2}+a\right)^{3 / 4}}
$$

Problem 218: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{3 / 4}}{x^{4}} \mathrm{~d} x
$$

Optimal(type 4, 126 leaves, 5 steps):

$$
\frac{b^{2} x}{2 a\left(b x^{2}+a\right)^{1 / 4}}-\frac{\left(b x^{2}+a\right)^{3 / 4}}{3 x^{3}}-\frac{b\left(b x^{2}+a\right)^{3 / 4}}{2 a x}-\frac{b^{3 / 2}\left(1+\frac{b x^{2}}{a}\right)^{1 / 4} \sqrt{\left.\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)\right)^{2}}}{\operatorname{EllipticE}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}
$$

Result(type 8, 47 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{3 / 4}\left(3 b x^{2}+2 a\right)}{6 a x^{3}}+\int \frac{b^{2}}{4 a\left(b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Problem 219: Unable to integrate problem.

$$
\int x^{4}\left(-b x^{2}+a\right)^{3 / 4} \mathrm{~d} x
$$

Optimal(type 4, 131 leaves, 5 steps):
$-\frac{4 a^{2} x\left(-b x^{2}+a\right)^{3 / 4}}{65 b^{2}}-\frac{2 a x^{3}\left(-b x^{2}+a\right)^{3 / 4}}{39 b}+\frac{2 x^{5}\left(-b x^{2}+a\right)^{3 / 4}}{13}$

$$
+\frac{8 a^{7 / 2}\left(1-\frac{b x^{2}}{a}\right)^{1 / 4} \sqrt{\cos \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}{65 \cos \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) b^{5 / 2}\left(-b x^{2}+a\right)^{1 / 4}}
$$

Result(type 8, 58 leaves):

$$
-\frac{2 x\left(-15 b^{2} x^{4}+5 a b x^{2}+6 a^{2}\right)\left(-b x^{2}+a\right)^{3 / 4}}{195 b^{2}}+\int \frac{4 a^{3}}{65 b^{2}\left(-b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Problem 220: Unable to integrate problem.

$$
\int \frac{\left(-b x^{2}+a\right)^{3 / 4}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 4, 93 leaves, 3 steps):

$$
-\frac{\left(-b x^{2}+a\right)^{3 / 4}}{x}-\frac{3\left(1-\frac{b x^{2}}{a}\right)^{1 / 4} \sqrt{\cos \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{a} \sqrt{b}}{\cos \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)\left(-b x^{2}+a\right)^{1 / 4}}
$$

Result(type 8, 31 leaves):

$$
-\frac{\left(-b x^{2}+a\right)^{3 / 4}}{x}+\int-\frac{3 b}{2\left(-b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Problem 221: Unable to integrate problem.

$$
\int \frac{x^{2}}{\left(b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 107 leaves, 4 steps):

$$
-\frac{4 a x}{5 b\left(b x^{2}+a\right)^{1 / 4}}+\frac{2 x\left(b x^{2}+a\right)^{3 / 4}}{5 b}+\frac{4 a^{3 / 2}\left(1+\frac{b x^{2}}{a}\right)^{1 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}{5 \cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) b^{3 / 2}\left(b x^{2}+a\right)^{1 / 4}}
$$

Result(type 8, 33 leaves):

$$
\frac{2 x\left(b x^{2}+a\right)^{3 / 4}}{5 b}+\int-\frac{2 a}{5 b\left(b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Problem 222: Unable to integrate problem.

$$
\int \frac{x^{2}}{\left(-b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 94 leaves, 3 steps):

$$
-\frac{2 x\left(-b x^{2}+a\right)^{3 / 4}}{5 b}+\frac{4 a^{3 / 2}\left(1-\frac{b x^{2}}{a}\right)^{1 / 4} \sqrt{\cos \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}{5 \cos \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) b^{3 / 2}\left(-b x^{2}+a\right)^{1 / 4}}
$$

Result(type 8, 35 leaves):

$$
-\frac{2 x\left(-b x^{2}+a\right)^{3 / 4}}{5 b}+\int \frac{2 a}{5 b\left(-b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Problem 223: Unable to integrate problem.

$$
\int \frac{1}{x^{2}\left(-b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 96 leaves, 3 steps):

$$
-\frac{\left(-b x^{2}+a\right)^{3 / 4}}{a x}-\frac{\left(1-\frac{b x^{2}}{a}\right)^{1 / 4} \sqrt{\cos \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{b}}{\cos \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)\left(-b x^{2}+a\right)^{1 / 4} \sqrt{a}}
$$

Result(type 8, 37 leaves):

$$
-\frac{\left(-b x^{2}+a\right)^{3 / 4}}{a x}+\int-\frac{b}{2 a\left(-b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Problem 224: Unable to integrate problem.

$$
\int \frac{x^{4}}{\left(b x^{2}+a\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 109 leaves, 4 steps):

$$
-\frac{4 a x\left(b x^{2}+a\right)^{1 / 4}}{7 b^{2}}+\frac{2 x^{3}\left(b x^{2}+a\right)^{1 / 4}}{7 b}+\frac{8 a^{5 / 2}\left(1+\frac{b x^{2}}{a}\right)^{3 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}{7 \cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) b^{5 / 2}\left(b x^{2}+a\right)^{3 / 4}}
$$

Result(type 8, 68 leaves):

$$
-\frac{2 x\left(-b x^{2}+2 a\right)\left(b x^{2}+a\right)^{1 / 4}}{7 b^{2}}+\frac{\left(\int \frac{4 a^{2}}{7 b^{2}\left(\left(b x^{2}+a\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{3}\right)^{1 / 4}}{\left(b x^{2}+a\right)^{3 / 4}}
$$

Problem 225: Unable to integrate problem.

$$
\int \frac{1}{x^{2}\left(b x^{2}+a\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 93 leaves, 3 steps):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 4}}{a x}-\frac{\left(1+\frac{b x^{2}}{a}\right)^{3 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{b}}{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)\left(b x^{2}+a\right)^{3 / 4} \sqrt{a}}
$$

Result(type 8, 58 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 4}}{a x}+\frac{\left(\int-\frac{b}{2 a\left(\left(b x^{2}+a\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{3}\right)^{1 / 4}}{\left(b x^{2}+a\right)^{3 / 4}}
$$

Problem 226: Unable to integrate problem.

$$
\int \frac{1}{x^{4}\left(b x^{2}+a\right)^{3 / 4}} d x
$$

Optimal(type 4, 111 leaves, 4 steps):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 4}}{3 a x^{3}}+\frac{5 b\left(b x^{2}+a\right)^{1 / 4}}{6 a^{2} x}+\frac{5 b^{3 / 2}\left(1+\frac{b x^{2}}{a}\right)^{3 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}{6 \cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) a^{3 / 2}\left(b x^{2}+a\right)^{3 / 4}}
$$

Result(type 8, 70 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 4}\left(-5 b x^{2}+2 a\right)}{6 a^{2} x^{3}}+\frac{\left(\int \frac{5 b^{2}}{12 a^{2}\left(\left(b x^{2}+a\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{3}\right)^{1 / 4}}{\left(b x^{2}+a\right)^{3 / 4}}
$$

Problem 227: Unable to integrate problem.

$$
\int \frac{x^{4}}{\left(-b x^{2}+a\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 113 leaves, 4 steps):

$$
-\frac{4 a x\left(-b x^{2}+a\right)^{1 / 4}}{7 b^{2}}-\frac{2 x^{3}\left(-b x^{2}+a\right)^{1 / 4}}{7 b}+\frac{8 a^{5 / 2}\left(1-\frac{b x^{2}}{a}\right)^{3 / 4} \sqrt{\cos \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}{7 \cos \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) b^{5 / 2}\left(-b x^{2}+a\right)^{3 / 4}}
$$

Result(type 8, 101 leaves):

$$
-\frac{2 x\left(b x^{2}+2 a\right)\left(-b x^{2}+a\right)^{1 / 4}\left(\left(-b x^{2}+a\right)^{3}\right)^{1 / 4}}{7 b^{2}\left(-\left(b x^{2}-a\right)^{3}\right)^{1 / 4}}+\frac{\left(\int \frac{4 a^{2}}{7 b^{2}\left(-\left(b x^{2}-a\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(-b x^{2}+a\right)^{3}\right)^{1 / 4}}{\left(-b x^{2}+a\right)^{3 / 4}}
$$

Problem 228: Unable to integrate problem.

$$
\int \frac{1}{x^{2}\left(-b x^{2}+a\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 95 leaves, 3 steps):

$$
-\frac{\left(-b x^{2}+a\right)^{1 / 4}}{a x}+\frac{\left(1-\frac{b x^{2}}{a}\right)^{3 / 4} \sqrt{\cos \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{b}}{\cos \left(\frac{\arcsin \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)\left(-b x^{2}+a\right)^{3 / 4} \sqrt{a}}
$$

Result(type 8, 92 leaves):

$$
-\frac{\left(-b x^{2}+a\right)^{1 / 4}\left(\left(-b x^{2}+a\right)^{3}\right)^{1 / 4}}{a x\left(-\left(b x^{2}-a\right)^{3}\right)^{1 / 4}}+\frac{\left(\int \frac{b}{2 a\left(-\left(b x^{2}-a\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(-b x^{2}+a\right)^{3}\right)^{1 / 4}}{\left(-b x^{2}+a\right)^{3 / 4}}
$$

Problem 229: Unable to integrate problem.

$$
\int \frac{x^{6}}{\left(b x^{2}+a\right)^{5 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 129 leaves, 5 steps):
$\frac{8 a^{2} x}{3 b^{3}\left(b x^{2}+a\right)^{1 / 4}}-\frac{4 a x^{3}}{9 b^{2}\left(b x^{2}+a\right)^{1 / 4}}+\frac{2 x^{5}}{9 b\left(b x^{2}+a\right)^{1 / 4}}$

$$
-\frac{16 a^{5 / 2}\left(1+\frac{b x^{2}}{a}\right)^{1 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}{3 \cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) b^{7 / 2}\left(b x^{2}+a\right)^{1 / 4}}
$$

Result(type 8, 66 leaves):

$$
-\frac{2 x\left(-b x^{2}+3 a\right)\left(b x^{2}+a\right)^{3 / 4}}{9 b^{3}}+\int \frac{a^{2}\left(5 b x^{2}+2 a\right)}{3 b^{4}\left(x^{2}+\frac{a}{b}\right)\left(b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Problem 230: Unable to integrate problem.

$$
\int \frac{1}{x^{4}\left(b x^{2}+a\right)^{5 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 111 leaves, 4 steps):

$$
-\frac{1}{3 a x^{3}\left(b x^{2}+a\right)^{1 / 4}}+\frac{7 b}{6 a^{2} x\left(b x^{2}+a\right)^{1 / 4}}+\frac{7 b^{3 / 2}\left(1+\frac{b x^{2}}{a}\right)^{1 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}{2 \cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) a^{5 / 2}\left(b x^{2}+a\right)^{1 / 4}}
$$

Result(type 8, 66 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{3 / 4}\left(-9 b x^{2}+2 a\right)}{6 a^{3} x^{3}}+\int-\frac{b\left(3 b x^{2}-a\right)}{4 a^{3}\left(x^{2}+\frac{a}{b}\right)\left(b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Problem 231: Result unnecessarily involves higher level functions.

$$
\int \frac{1}{\left(3 x^{2}+2\right)^{1 / 4}} d x
$$

Optimal(type 4, 61 leaves, 2 steps):

$$
\frac{2 x}{\left(3 x^{2}+2\right)^{1 / 4}}-\frac{22^{1 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{6}}{2}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{6}}{2}\right)}{2}\right), \sqrt{2}\right) \sqrt{3}}{3 \cos \left(\frac{\arctan \left(\frac{x \sqrt{6}}{2}\right)}{2}\right)}
$$

Result(type 5, 17 leaves):

$$
\frac{2^{3 / 4} x \text { hypergeom }\left(\left[\frac{1}{4}, \frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{2}
$$

Problem 232: Result unnecessarily involves higher level functions.

$$
\int \frac{1}{x^{2}\left(3 x^{2}+2\right)^{1 / 4}} d x
$$

Optimal(type 4, 75 leaves, 3 steps):

$$
\frac{3 x}{2\left(3 x^{2}+2\right)^{1 / 4}}-\frac{\left(3 x^{2}+2\right)^{3 / 4}}{2 x}-\frac{2^{1 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{6}}{2}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{6}}{2}\right)}{2}\right), \sqrt{2}\right) \sqrt{3}}{2 \cos \left(\frac{\arctan \left(\frac{x \sqrt{6}}{2}\right)}{2}\right)}
$$

Result(type 5, 32 leaves):

$$
-\frac{\left(3 x^{2}+2\right)^{3 / 4}}{2 x}+\frac{32^{3 / 4} x \text { hypergeom }\left(\left[\frac{1}{4}, \frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{8}
$$

Problem 233: Result unnecessarily involves higher level functions.

$$
\int \frac{1}{x^{6}\left(-3 x^{2}+2\right)^{1 / 4}} d x
$$

Optimal(type 4, 91 leaves, 4 steps):

$$
-\frac{\left(-3 x^{2}+2\right)^{3 / 4}}{10 x^{5}}-\frac{7\left(-3 x^{2}+2\right)^{3 / 4}}{40 x^{3}}-\frac{63\left(-3 x^{2}+2\right)^{3 / 4}}{160 x}-\frac{63 \sqrt{\cos \left(\frac{\left.\arcsin \left(\frac{x \sqrt{6}}{2}\right)\right)^{2}}{2}\right)} \text { EllipticE }\left(\sin \left(\frac{\arcsin \left(\frac{x \sqrt{6}}{2}\right)}{2}\right), \sqrt{2}\right) \sqrt{3} 2^{1 / 4}}{160 \cos \left(\frac{\arcsin \left(\frac{x \sqrt{6}}{2}\right)}{2}\right)}
$$

Result(type 5, 49 leaves):

$$
\frac{189 x^{6}-42 x^{4}-8 x^{2}-32}{160 x^{5}\left(-3 x^{2}+2\right)^{1 / 4}}-\frac{1892^{3 / 4} x \text { hypergeom }\left(\left[\frac{1}{4}, \frac{1}{2}\right],\left[\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{640}
$$

Problem 234: Result unnecessarily involves higher level functions.

$$
\int \frac{x^{4}}{\left(3 x^{2}+2\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 75 leaves, 3 steps):

$$
-\frac{8 x\left(3 x^{2}+2\right)^{1 / 4}}{63}+\frac{2 x^{3}\left(3 x^{2}+2\right)^{1 / 4}}{21}+\frac{162^{3 / 4} \sqrt{\cos \left(\frac{\left.\arctan \left(\frac{x \sqrt{6}}{2}\right)\right)^{2}}{2}\right)^{\operatorname{EllipticF}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{6}}{2}\right)}{2}\right), \sqrt{2}\right) \sqrt{3}}} 1189 \cos \left(\frac{\arctan \left(\frac{x \sqrt{6}}{2}\right)}{2}\right)}{\text { ( } \left.\frac{1}{2}\right)}
$$

Result(type 5, 37 leaves):

$$
\frac{2 x\left(3 x^{2}-4\right)\left(3 x^{2}+2\right)^{1 / 4}}{63}+\frac{82^{1 / 4} x \text { hypergeom }\left(\left[\frac{1}{2}, \frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{63}
$$

Problem 235: Result unnecessarily involves higher level functions.

$$
\int \frac{x^{2}}{\left(3 x^{2}+2\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 61 leaves, 2 steps):

$$
\frac{2 x\left(3 x^{2}+2\right)^{1 / 4}}{9}-\frac{42^{3 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{6}}{2}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{6}}{2}\right)}{2}\right), \sqrt{2}\right) \sqrt{3}}{27 \cos \left(\frac{\arctan \left(\frac{x \sqrt{6}}{2}\right)}{2}\right)}
$$

Result(type 5, 30 leaves):

$$
\frac{2 x\left(3 x^{2}+2\right)^{1 / 4}}{9}-\frac{22^{1 / 4} x \text { hypergeom }\left(\left[\frac{1}{2}, \frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{9}
$$

Problem 236: Result unnecessarily involves higher level functions.

$$
\int \frac{1}{x^{2}\left(3 x^{2}+2\right)^{3 / 4}} d x
$$

Optimal(type 4, 63 leaves, 2 steps):

$$
-\frac{\left(3 x^{2}+2\right)^{1 / 4}}{2 x}-\frac{2^{3 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{6}}{2}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{6}}{2}\right)}{2}\right), \sqrt{2}\right) \sqrt{3}}{4 \cos \left(\frac{\arctan \left(\frac{x \sqrt{6}}{2}\right)}{2}\right)}
$$

Result(type 5, 32 leaves):

$$
-\frac{\left(3 x^{2}+2\right)^{1 / 4}}{2 x}-\frac{32^{1 / 4} x \text { hypergeom }\left(\left[\frac{1}{2}, \frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{8}
$$

Problem 237: Result unnecessarily involves higher level functions.

$$
\int \frac{x^{6}}{\left(3 x^{2}-2\right)^{1 / 4}} d x
$$

Optimal(type 4, 290 leaves, 7 steps):
$\frac{32 x\left(3 x^{2}-2\right)^{3 / 4}}{1053}+\frac{40 x^{3}\left(3 x^{2}-2\right)^{3 / 4}}{1053}+\frac{2 x^{5}\left(3 x^{2}-2\right)^{3 / 4}}{39}+\frac{128 x\left(3 x^{2}-2\right)^{1 / 4}}{1053\left(\sqrt{2}+\sqrt{3 x^{2}-2}\right)}$

$$
\begin{aligned}
& -\frac{1}{3159 \cos \left(2 \arctan \left(\frac{\left(3 x^{2}-2\right)^{1 / 4} 2^{3} / 4}{2}\right)\right) x}\left(1 2 8 2 ^ { 1 / 4 } \sqrt { \operatorname { c o s } ( 2 \operatorname { a r c t a n } ( \frac { ( 3 x ^ { 2 } - 2 ) ^ { 1 / 4 } 2 ^ { 3 / 4 } } { 2 } ) ) ^ { 2 } } \text { EllipticE } \left(\sin \left(2 \arctan \left(\frac{\left(3 x^{2}-2\right)^{1 / 4} 2^{3} / 4}{2}\right)\right)\right.\right. \\
& \left.\left.\frac{\sqrt{2}}{2}\right)\left(\sqrt{2}+\sqrt{3 x^{2}-2}\right) \sqrt{\frac{x^{2}}{\left(\sqrt{2}+\sqrt{3 x^{2}-2}\right)^{2}}} \sqrt{3}\right) \\
& +\frac{3159 \cos \left(2 \arctan \left(\frac{\left(3 x^{2}-2\right)^{1 / 4} 2^{3} / 4}{2}\right)\right) x}{\left(6 4 2 ^ { 1 / 4 } \sqrt { \operatorname { c o s } ( 2 \operatorname { a r c t a n } ( \frac { ( 3 x ^ { 2 } - 2 ) ^ { 1 / 4 } 2 ^ { 3 / 4 } } { 2 } ) ) ^ { 2 } } \operatorname { E l l i p t i c F } \left(\sin \left(2 \arctan \left(\frac{\left(3 x^{2}-2\right)^{1 / 4} 2^{3} / 4}{2}\right)\right)\right.\right.} \\
& \left.\left.\frac{\sqrt{2}}{2}\right)\left(\sqrt{2}+\sqrt{3 x^{2}-2}\right) \sqrt{\frac{x^{2}}{\left(\sqrt{2}+\sqrt{3 x^{2}-2}\right)^{2}}} \sqrt{3}\right)
\end{aligned}
$$

Result(type 5, 64 leaves):

$$
\frac{2 x\left(27 x^{4}+20 x^{2}+16\right)\left(3 x^{2}-2\right)^{3 / 4}}{1053}+\frac{322^{3 / 4}\left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{1 / 4} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right],\left[\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{1053 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{1 / 4}}
$$

Problem 238: Result unnecessarily involves higher level functions.

$$
\int \frac{x^{2}}{\left(-3 x^{2}-2\right)^{1 / 4}} d x
$$

$$
\begin{aligned}
& \text { Optimal (type 4, } 264 \text { leaves, } 5 \text { steps) : } \\
& -\frac{2 x\left(-3 x^{2}-2\right)^{3 / 4}}{15}-\frac{8 x\left(-3 x^{2}-2\right)^{1 / 4}}{15\left(\sqrt{2}+\sqrt{-3 x^{2}-2}\right)} \\
& -\frac{1}{45 \cos \left(2 \arctan \left(\frac{\left(-3 x^{2}-2\right)^{1 / 4} 2^{3} / 4}{2}\right)\right) x}\left(82^{1 / 4} \sqrt{\cos \left(2 \operatorname { a r c t a n } \left(\frac { ( - 3 x ^ { 2 } - 2 ) ^ { 1 / 4 } 2 ^ { 3 / 4 } ) ) ^ { 2 } } { 2 } \operatorname { E l l i p t i c E } \left(\sin \left(2 \arctan \left(\frac{\left(-3 x^{2}-2\right)^{1 / 4} 2^{3} / 4}{2}\right)\right),\right.\right.\right.}\right. \\
& \left.\frac{\sqrt{2}}{2}\right)\left(\sqrt{2}+\sqrt{-3 x^{2}-2}\right) \sqrt{\left(\sqrt{2}+\sqrt{-3 x^{2}-2}\right)^{2}} \\
& \quad+\frac{x^{2}}{45 \cos \left(2 \arctan \left(\frac{\left(-3 x^{2}-2\right)^{1 / 4} 2^{3} / 4}{2}\right)\right.}\left(4 2 ^ { 1 / 4 } \sqrt { \operatorname { c o s } ( 2 \operatorname { a r c t a n } ( \frac { ( - 3 x ^ { 2 } - 2 ) ^ { 1 / 4 } 2 ^ { 3 / 4 } } { 2 } ) ) ^ { 2 } } \operatorname { E l l i p t i c F } \left(\sin \left(2 \arctan \left(\frac{\left(-3 x^{2}-2\right)^{1 / 4} 2^{3} / 4}{2}\right)\right),\right.\right.
\end{aligned}
$$

$$
\left.\left.\frac{\sqrt{2}}{2}\right)\left(\sqrt{2}+\sqrt{-3 x^{2}-2}\right) \sqrt{-\frac{x^{2}}{\left(\sqrt{2}+\sqrt{-3 x^{2}-2}\right)^{2}}} \sqrt{3}\right)
$$

Result(type 5, 40 leaves):

$$
\frac{2 x\left(3 x^{2}+2\right)}{15\left(-3 x^{2}-2\right)^{1 / 4}}+\frac{2(-1)^{3 / 4} 2^{3 / 4} x \text { hypergeom }\left(\left[\frac{1}{4}, \frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{15}
$$

Problem 239: Result unnecessarily involves higher level functions.

$$
\int \frac{1}{\left(3 x^{2}-2\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 111 leaves, 2 steps):
$\frac{\sqrt{\cos \left(2 \arctan \left(\frac{\left(3 x^{2}-2\right)^{1 / 4} 2^{3 / 4}}{2}\right)\right)^{2}} \operatorname{EllipticF}\left(\sin \left(2 \arctan \left(\frac{\left(3 x^{2}-2\right)^{1 / 4} 2^{3 / 4}}{2}\right)\right), \frac{\sqrt{2}}{2}\right)\left(\sqrt{2}+\sqrt{3 x^{2}-2}\right) \sqrt{\left(\sqrt{2}+\sqrt{3 x^{2}-2}\right)^{2}} 2^{3 / 4} \sqrt{3}}{6 \cos \left(2 \arctan \left(\frac{\left(3 x^{2}-2\right)^{1 / 4} 2^{3 / 4}}{2}\right)\right) x}$
Result(type 5, 39 leaves):

$$
\frac{2^{1 / 4}\left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{3 / 4} x \text { hypergeom }\left(\left[\frac{1}{2}, \frac{3}{4}\right],\left[\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{2 \text { signum }\left(-1+\frac{3 x^{2}}{2}\right)^{3 / 4}}
$$

Problem 240: Unable to integrate problem.

$$
\int(c x)^{7 / 2}\left(b x^{2}+a\right)^{1 / 4} \mathrm{~d} x
$$

Optimal(type 4, 149 leaves, 8 steps):
$\frac{a c(c x)^{5 / 2}\left(b x^{2}+a\right)^{1 / 4}}{30 b}+\frac{(c x)^{9 / 2}\left(b x^{2}+a\right)^{1 / 4}}{5 c}$

$$
-\frac{a^{5 / 2} c^{2}\left(1+\frac{a}{b x^{2}}\right)^{3 / 4}(c x)^{3 / 2} \sqrt{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}{12 \cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) b^{3 / 2}\left(b x^{2}+a\right)^{3 / 4}}-\frac{a^{2} c^{3}\left(b x^{2}+a\right)^{1 / 4} \sqrt{c x}}{12 b^{2}}
$$

Result(type 8, 111 leaves):

$$
-\frac{\left(-12 b^{2} x^{4}-2 a b x^{2}+5 a^{2}\right)\left(b x^{2}+a\right)^{1 / 4} c^{3} \sqrt{c x}}{60 b^{2}}+\frac{\left(\int \frac{a^{3}}{24 b^{2}\left(c^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right) c^{3} \sqrt{c x}\left(c^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}}{x\left(b x^{2}+a\right)^{3 / 4}}
$$

Problem 241: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{1 / 4}}{\sqrt{c x}} \mathrm{~d} x
$$

Optimal(type 4, 102 leaves, 6 steps):

$$
-\frac{\left(1+\frac{a}{b x^{2}}\right)^{3 / 4}(c x)^{3 / 2} \sqrt{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{a} \sqrt{b}}{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) c^{2}\left(b x^{2}+a\right)^{3 / 4}}+\frac{\left(b x^{2}+a\right)^{1 / 4} \sqrt{c x}}{c}
$$

Result(type 8, 73 leaves):

$$
\frac{\left(b x^{2}+a\right)^{1 / 4} x}{\sqrt{c x}}+\frac{\left(\int \frac{a}{2\left(c^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(c^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}}{\sqrt{c x}\left(b x^{2}+a\right)^{3 / 4}}
$$

Problem 242: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{1 / 4}}{(c x)^{13 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 151 leaves, 8 steps):

$$
-\frac{2\left(b x^{2}+a\right)^{1 / 4}}{11 c(c x)^{11 / 2}}-\frac{2 b\left(b x^{2}+a\right)^{1 / 4}}{77 a c^{3}(c x)^{7 / 2}}+\frac{4 b^{2}\left(b x^{2}+a\right)^{1 / 4}}{77 a^{2} c^{5}(c x)^{3 / 2}}
$$

$$
-\frac{8 b^{7 / 2}\left(1+\frac{a}{b x^{2}}\right)^{3 / 4}(c x)^{3 / 2} \sqrt{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}{77 \cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) a^{5 / 2} c^{8}\left(b x^{2}+a\right)^{3 / 4}}
$$

Result(type 8, 110 leaves):

$$
-\frac{2\left(b x^{2}+a\right)^{1 / 4}\left(-2 b^{2} x^{4}+a b x^{2}+7 a^{2}\right)}{77 x^{5} a^{2} c^{6} \sqrt{c x}}+\frac{\left(\int \frac{4 b^{3}}{77 a^{2}\left(c^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(c^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}}{c^{6} \sqrt{c x}\left(b x^{2}+a\right)^{3 / 4}}
$$

Problem 243: Unable to integrate problem.

$$
\int(c x)^{5 / 2}\left(-b x^{2}+a\right)^{1 / 4} \mathrm{~d} x
$$

Optimal(type 3, 246 leaves, 13 steps):

$$
\begin{aligned}
& -\frac{a c(c x)^{3 / 2}\left(-b x^{2}+a\right)^{1 / 4}}{16 b}+\frac{(c x)^{7 / 2}\left(-b x^{2}+a\right)^{1 / 4}}{4 c}+\frac{3 a^{2} c^{5 / 2} \arctan \left(-1+\frac{b^{1 / 4} \sqrt{2} \sqrt{c x}}{\left(-b x^{2}+a\right)^{1 / 4} \sqrt{c}}\right) \sqrt{2}}{64 b^{7 / 4}} \\
& +\frac{3 a^{2} c^{5 / 2} \arctan \left(1+\frac{b^{1 / 4} \sqrt{2} \sqrt{c x}}{\left(-b x^{2}+a\right)^{1 / 4} \sqrt{c}}\right) \sqrt{2}}{64 b^{7 / 4}}+\frac{3 a^{2} c^{5 / 2} \ln \left(\sqrt{c}-\frac{b^{1 / 4} \sqrt{2} \sqrt{c x}}{\left(-b x^{2}+a\right)^{1 / 4}}+\frac{x \sqrt{b} \sqrt{c}}{\sqrt{-b x^{2}+a}}\right) \sqrt{2}}{128 b^{7 / 4}} \\
& -\frac{3 a^{2} c^{5 / 2} \ln \left(\sqrt{c}+\frac{b^{1 / 4} \sqrt{2} \sqrt{c x}}{\left(-b x^{2}+a\right)^{1 / 4}}+\frac{x \sqrt{b} \sqrt{c}}{\sqrt{-b x^{2}+a}}\right) \sqrt{2}}{128 b^{7 / 4}}
\end{aligned}
$$

Result(type 8, 162 leaves):

Problem 245: Unable to integrate problem.

$$
\int \frac{1}{\sqrt{c x}\left(b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Optimal(type 3, 59 leaves, 5 steps):

$$
\frac{\arctan \left(\frac{b^{1 / 4} \sqrt{c x}}{\left(b x^{2}+a\right)^{1 / 4} \sqrt{c}}\right)}{b^{1 / 4} \sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{b^{1 / 4} \sqrt{c x}}{\left(b x^{2}+a\right)^{1 / 4} \sqrt{c}}\right)}{b^{1 / 4} \sqrt{c}}
$$

Result(type 8, 17 leaves):

$$
\int \frac{1}{\sqrt{c x}\left(b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Problem 247: Unable to integrate problem.

$$
\int \frac{1}{(c x)^{3 / 2}\left(b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 103 leaves, 4 steps):
$-\frac{2}{c\left(b x^{2}+a\right)^{1 / 4} \sqrt{c x}}+\frac{2\left(1+\frac{a}{b x^{2}}\right)^{1 / 4} \sqrt{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)^{2}}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{b} \sqrt{c x}}{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) c^{2}\left(b x^{2}+a\right)^{1 / 4} \sqrt{a}}$
Result(type 8, 82 leaves):

$$
-\frac{2\left(b x^{2}+a\right)^{3 / 4}}{a c \sqrt{c x}}+\frac{\left(\int \frac{2 b x}{a\left(\left(b x^{2}+a\right) c^{2} x^{2}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right) c^{2} x^{2}\right)^{1 / 4}}{c \sqrt{c x}\left(b x^{2}+a\right)^{1 / 4}}
$$

Problem 248: Unable to integrate problem.

$$
\int \frac{(c x)^{3 / 2}}{\left(-b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Optimal(type 3, 217 leaves, 12 steps):

$$
\left.\begin{array}{l}
a c^{3 / 2} \arctan \left(-1+\frac{b^{1 / 4} \sqrt{2} \sqrt{c x}}{\left(-b x^{2}+a\right)^{1 / 4} \sqrt{c}}\right) \sqrt{2} \\
8 b^{5 / 4} \\
\quad+\frac{a c^{3 / 2} \arctan \left(1+\frac{b^{1 / 4} \sqrt{2} \sqrt{c x}}{\left(-b x^{2}+a\right)^{1 / 4} \sqrt{c}}\right) \sqrt{2}}{8 b^{5 / 4}}-\frac{a c^{3 / 2} \ln \left(\sqrt{c}-\frac{b^{1 / 4} \sqrt{2} \sqrt{c x}}{\left(-b x^{2}+a\right)^{1 / 4}}+\frac{x \sqrt{b} \sqrt{c}}{\sqrt{-b x^{2}+a}}\right) \sqrt{2}}{16 b^{5 / 4}} \\
\quad a c^{3 / 2} \ln \left(\sqrt{c}+\frac{b^{1 / 4} \sqrt{2} \sqrt{c x}}{\left(-b x^{2}+a\right)^{1 / 4}}+\frac{x \sqrt{b} \sqrt{c}}{\sqrt{-b x^{2}+a}}\right) \sqrt{2} \\
16 b^{5 / 4}
\end{array} \frac{c\left(-b x^{2}+a\right)^{3 / 4} \sqrt{c x}}{2 b}\right) .
$$

Result(type 8, 84 leaves):

$$
-\frac{c\left(-b x^{2}+a\right)^{3 / 4} \sqrt{c x}}{2 b}+\frac{\left(\int \frac{a}{4 b\left(c^{2} x^{2}\left(-b x^{2}+a\right)\right)^{1 / 4}} \mathrm{~d} x\right) c \sqrt{c x}\left(c^{2} x^{2}\left(-b x^{2}+a\right)\right)^{1 / 4}}{x\left(-b x^{2}+a\right)^{1 / 4}}
$$

Problem 250: Unable to integrate problem.

$$
\int \frac{(c x)^{5 / 2}}{\left(-b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 131 leaves, 5 steps):
$-\frac{c(c x)^{3 / 2}\left(-b x^{2}+a\right)^{3 / 4}}{3 b}-\frac{a c^{3}\left(-b x^{2}+a\right)^{3 / 4}}{2 b^{2} \sqrt{c x}}+\frac{a^{3 / 2} c^{2}\left(1-\frac{a}{b x^{2}}\right)^{1 / 4} \sqrt{\cos \left(\frac{\operatorname{arccsc}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)} \operatorname{EllipticE}\left(\sin \left(\frac{\operatorname{arccsc}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{c x}}{2 \cos \left(\frac{\operatorname{arccsc}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) b^{3 / 2}\left(-b x^{2}+a\right)^{1 / 4}}$
Result(type 8, 90 leaves):

$$
-\frac{x\left(-b x^{2}+a\right)^{3 / 4} c^{2} \sqrt{c x}}{3 b}+\frac{\left(\int \frac{a x}{2 b\left(c^{2} x^{2}\left(-b x^{2}+a\right)\right)^{1 / 4}} \mathrm{~d} x\right) c^{2} \sqrt{c x}\left(c^{2} x^{2}\left(-b x^{2}+a\right)\right)^{1 / 4}}{x\left(-b x^{2}+a\right)^{1 / 4}}
$$

Problem 251: Unable to integrate problem.

$$
\int \frac{1}{(c x)^{3 / 2}\left(-b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 85 leaves, 3 steps):

$$
-\frac{2\left(1-\frac{a}{b x^{2}}\right)^{1 / 4} \sqrt{\cos \left(\frac{\operatorname{arccsc}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)} \operatorname{EllipticE}\left(\sin \left(\frac{\operatorname{arccsc}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{b} \sqrt{c x}}{\cos \left(\frac{\operatorname{arccsc}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) c^{2}\left(-b x^{2}+a\right)^{1 / 4} \sqrt{a}}
$$

Result(type 8, 86 leaves):

$$
-\frac{2\left(-b x^{2}+a\right)^{3 / 4}}{a c \sqrt{c x}}+\frac{\left(\int-\frac{2 b x}{a\left(c^{2} x^{2}\left(-b x^{2}+a\right)\right)^{1 / 4}} \mathrm{~d} x\right)\left(c^{2} x^{2}\left(-b x^{2}+a\right)\right)^{1 / 4}}{c \sqrt{c x}\left(-b x^{2}+a\right)^{1 / 4}}
$$

Problem 252: Unable to integrate problem.

$$
\int \frac{1}{(c x)^{9 / 2}\left(-b x^{2}+a\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 133 leaves, 7 steps):

$$
-\frac{2\left(-b x^{2}+a\right)^{1 / 4}}{7 a c(c x)^{7 / 2}}-\frac{4 b\left(-b x^{2}+a\right)^{1 / 4}}{7 a^{2} c^{3}(c x)^{3 / 2}}-\frac{8 b^{5 / 2}\left(1-\frac{a}{b x^{2}}\right)^{3 / 4}(c x)^{3 / 2} \sqrt{\left.\cos \left(\frac{\operatorname{arccsc}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\operatorname{arccsc}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}{7 \cos \left(\frac{\operatorname{arccsc}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) a^{5 / 2} c^{6}\left(-b x^{2}+a\right)^{3 / 4}}
$$

Result(type 8, 160 leaves):

$$
-\frac{2\left(-b x^{2}+a\right)^{1 / 4}\left(2 b x^{2}+a\right)\left(\left(-b x^{2}+a\right)^{3}\right)^{1 / 4}}{7 a^{2} x^{3} \sqrt{c x}\left(-\left(b x^{2}-a\right)^{3}\right)^{1 / 4} c^{4}}+\frac{\left(\int \frac{4 b^{2}}{7 a^{2}\left(-c^{2} x^{2}\left(b x^{2}-a\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(-b x^{2}+a\right)^{3}\right)^{1 / 4}\left(-c^{2} x^{2}\left(b x^{2}-a\right)^{3}\right)^{1 / 4}}{\sqrt{c x}\left(-b x^{2}+a\right)^{3 / 4}\left(-\left(b x^{2}-a\right)^{3}\right)^{1 / 4} c^{4}}
$$

Problem 255: Unable to integrate problem.

$$
\int \frac{(c x)^{5 / 2}}{\left(b x^{2}+a\right)^{5 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 103 leaves, 4 steps):

$$
\frac{c(c x)^{3 / 2}}{b\left(b x^{2}+a\right)^{1 / 4}}+\frac{3 c^{2}\left(1+\frac{a}{b x^{2}}\right)^{1 / 4} \sqrt{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{a} \sqrt{c x}}{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) b^{3 / 2}\left(b x^{2}+a\right)^{1 / 4}}
$$

Result(type 8, 17 leaves):

$$
\int \frac{(c x)^{5 / 2}}{\left(b x^{2}+a\right)^{5 / 4}} \mathrm{~d} x
$$

Problem 256: Unable to integrate problem.

$$
\int \frac{\sqrt{c x}}{\left(b x^{2}+a\right)^{5 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 80 leaves, 3 steps):

$$
-\frac{2\left(1+\frac{a}{b x^{2}}\right)^{1 / 4} \sqrt{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)} \operatorname{EllipticE}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{c x}}{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)\left(b x^{2}+a\right)^{1 / 4} \sqrt{a} \sqrt{b}}
$$

Result(type 8, 17 leaves):

$$
\int \frac{\sqrt{c x}}{\left(b x^{2}+a\right)^{5 / 4}} \mathrm{~d} x
$$

Problem 257: Unable to integrate problem.

$$
\int \frac{1}{(c x)^{3 / 2}\left(b x^{2}+a\right)^{5 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 106 leaves, 4 steps):

$$
-\frac{2}{a c\left(b x^{2}+a\right)^{1 / 4} \sqrt{c x}}+\frac{4\left(1+\frac{a}{b x^{2}}\right)^{1 / 4} \sqrt{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{b} \sqrt{c x}}{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) a^{3 / 2} c^{2}\left(b x^{2}+a\right)^{1 / 4}}
$$

Result(type 8, 99 leaves):

$$
-\frac{2\left(b x^{2}+a\right)^{3 / 4}}{a^{2} c \sqrt{c x}}+\frac{\left(\int \frac{x\left(2 b x^{2}+a\right)}{a^{2}\left(x^{2}+\frac{a}{b}\right)\left(\left(b x^{2}+a\right) c^{2} x^{2}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right) c^{2} x^{2}\right)^{1 / 4}}{c \sqrt{c x}\left(b x^{2}+a\right)^{1 / 4}}
$$

Problem 258: Unable to integrate problem.

$$
\int \frac{1}{(c x)^{7 / 2}\left(b x^{2}+a\right)^{5 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 129 leaves, 5 steps):
$-\frac{2}{5 a c(c x)^{5 / 2}\left(b x^{2}+a\right)^{1 / 4}}+\frac{12 b}{5 a^{2} c^{3}\left(b x^{2}+a\right)^{1 / 4} \sqrt{c x}}$

$$
-\frac{24 b^{3 / 2}\left(1+\frac{a}{b x^{2}}\right)^{1 / 4} \sqrt{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{c x}}{5 \cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) a^{5 / 2} c^{4}\left(b x^{2}+a\right)^{1 / 4}}
$$

Result(type 8, 114 leaves):

$$
-\frac{2\left(b x^{2}+a\right)^{3 / 4}\left(-7 b x^{2}+a\right)}{5 a^{3} x^{2} c^{3} \sqrt{c x}}+\frac{\left(\int-\frac{b x\left(14 b x^{2}+9 a\right)}{5 a^{3}\left(x^{2}+\frac{a}{b}\right)\left(\left(b x^{2}+a\right) c^{2} x^{2}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right) c^{2} x^{2}\right)^{1 / 4}}{c^{3} \sqrt{c x}\left(b x^{2}+a\right)^{1 / 4}}
$$

Problem 259: Unable to integrate problem.

$$
\int \frac{1}{(c x)^{11 / 2}\left(b x^{2}+a\right)^{5 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 154 leaves, 6 steps):

$$
\begin{aligned}
& -\frac{2}{9 a c(c x)^{9 / 2}\left(b x^{2}+a\right)^{1 / 4}}+\frac{4 b}{9 a^{2} c^{3}(c x)^{5 / 2}\left(b x^{2}+a\right)^{1 / 4}}-\frac{8 b^{2}}{3 a^{3} c^{5}\left(b x^{2}+a\right)^{1 / 4} \sqrt{c x}} \\
& \left.+\frac{16 b^{5 / 2}\left(1+\frac{a}{b x^{2}}\right)^{1 / 4} \sqrt{\left.\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{c x}}{\left(\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)\right.}\right) a^{7 / 2} c^{6}\left(b x^{2}+a\right)^{1 / 4}
\end{aligned}
$$

Result(type 8, 127 leaves):

$$
-\frac{2\left(b x^{2}+a\right)^{3 / 4}\left(15 b^{2} x^{4}-3 a b x^{2}+a^{2}\right)}{9 a^{4} x^{4} c^{5} \sqrt{c x}}+\frac{\left(\int \frac{b^{2} x\left(10 b x^{2}+7 a\right)}{3 a^{4}\left(x^{2}+\frac{a}{b}\right)\left(\left(b x^{2}+a\right) c^{2} x^{2}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right) c^{2} x^{2}\right)^{1 / 4}}{c^{5} \sqrt{c x}\left(b x^{2}+a\right)^{1 / 4}}
$$

Problem 260: Unable to integrate problem.

$$
\int \frac{1}{(c x)^{3 / 4}\left(b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Optimal(type 5, 46 leaves, 2 steps):

$$
\frac{4(c x)^{1 / 4}\left(1+\frac{b x^{2}}{a}\right)^{1 / 4} \text { hypergeom }\left(\left[\frac{1}{8}, \frac{1}{4}\right],\left[\frac{9}{8}\right],-\frac{b x^{2}}{a}\right)}{c\left(b x^{2}+a\right)^{1 / 4}}
$$

Result(type 8, 17 leaves):

$$
\int \frac{1}{(c x)^{3 / 4}\left(b x^{2}+a\right)^{1 / 4}} \mathrm{~d} x
$$

Problem 261: Unable to integrate problem.

$$
\int \frac{(c x)^{5 / 4}}{\left(b x^{2}+a\right)^{7 / 4}} \mathrm{~d} x
$$

Optimal(type 5, 49 leaves, 2 steps):

$$
\frac{4(c x)^{9 / 4}\left(1+\frac{b x^{2}}{a}\right)^{3 / 4} \text { hypergeom }\left(\left[\frac{9}{8}, \frac{7}{4}\right],\left[\frac{17}{8}\right],-\frac{b x^{2}}{a}\right)}{9 a c\left(b x^{2}+a\right)^{3 / 4}}
$$

Result(type 8, 17 leaves):

$$
\int \frac{(c x)^{5 / 4}}{\left(b x^{2}+a\right)^{7 / 4}} \mathrm{~d} x
$$

Problem 262: Unable to integrate problem.

$$
\int \frac{1}{(c x)^{3 / 4}\left(b x^{2}+a\right)^{7 / 4}} \mathrm{~d} x
$$

Optimal(type 5, 49 leaves, 2 steps):

$$
\frac{4(c x)^{1 / 4}\left(1+\frac{b x^{2}}{a}\right)^{3 / 4} \text { hypergeom }\left(\left[\frac{1}{8}, \frac{7}{4}\right],\left[\frac{9}{8}\right],-\frac{b x^{2}}{a}\right)}{a c\left(b x^{2}+a\right)^{3 / 4}}
$$

Result(type 8, 17 leaves):

$$
\int \frac{1}{(c x)^{3 / 4}\left(b x^{2}+a\right)^{7 / 4}} d x
$$

Problem 263: Unable to integrate problem.

$$
\int x^{2}\left(b x^{2}+a\right)^{1 / 6} \mathrm{~d} x
$$

Optimal(type 4, 247 leaves, 5 steps):

$$
\begin{aligned}
& \left.\frac{3 a x\left(b x^{2}+a\right)^{1 / 6}}{40 b}+\frac{3 x^{3}\left(b x^{2}+a\right)^{1 / 6}}{10}-\frac{1}{\left.40 b^{2} x\left(\frac{a}{b x^{2}+a}\right)^{1 / 3} \sqrt{\frac{-1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}}\right]^{33^{3} / 4} a^{2}\left(b x^{2}+a\right)^{1 / 6}(1} 1\left(\frac{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\sqrt{3}}{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) / \frac{1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}\right)
\end{aligned}
$$

Result(type 8, 66 leaves):

$$
\frac{3 x\left(4 b x^{2}+a\right)\left(b x^{2}+a\right)^{1 / 6}}{40 b}+\frac{\left(\int-\frac{3 a^{2}}{40 b\left(\left(b x^{2}+a\right)^{5}\right)^{1 / 6}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{5}\right)^{1 / 6}}{\left(b x^{2}+a\right)^{5 / 6}}
$$

Problem 264: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{1 / 6}}{x^{2}} \mathrm{~d} x
$$

Optimal(type 4, 225 leaves, 4 steps):

$$
\begin{aligned}
& -\frac{\left(b x^{2}+a\right)^{1 / 6}}{x}+\frac{1}{3 x\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}\left(\frac{-1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}\right.}\left(\left(b x^{2}+a\right)^{1 / 6}(1\right. \\
& \left.-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}\right) \text { EllipticF }\left(\frac{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\sqrt{3}}{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) / \frac{1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\left(\frac{a}{b x^{2}+a}\right)^{2 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}
\end{aligned}
$$

Result(type 8, 52 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 6}}{x}+\frac{\left(\int \frac{b}{3\left(\left(b x^{2}+a\right)^{5}\right)^{1 / 6}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{5}\right)^{1 / 6}}{\left(b x^{2}+a\right)^{5 / 6}}
$$

Problem 265: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{1 / 6}}{x^{4}} \mathrm{~d} x
$$

Optimal(type 4, 247 leaves, 5 steps):

$$
\begin{aligned}
& -\frac{\left(b x^{2}+a\right)^{1 / 6}}{3 x^{3}}-\frac{b\left(b x^{2}+a\right)^{1 / 6}}{9 a x}-\frac{1}{\left.27 a x\left(\frac{a}{b x^{2}+a}\right)^{1 / 3} / \sqrt{\frac{-1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}}\right)} 2\left(2 b\left(b x^{2}+a\right)^{1 / 6}(1\right. \\
& \left.-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}\right) \text { EllipticF }\left(\frac{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\sqrt{3}}{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) / \frac{1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\left(\frac{a}{b x^{2}+a}\right)^{2 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}
\end{aligned}
$$

Result(type 8, 69 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 6}\left(b x^{2}+3 a\right)}{9 x^{3} a}+\frac{\left(\int-\frac{2 b^{2}}{27 a\left(\left(b x^{2}+a\right)^{5}\right)^{1 / 6}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{5}\right)^{1 / 6}}{\left(b x^{2}+a\right)^{5 / 6}}
$$

Problem 266: Unable to integrate problem.

$$
\int \frac{1}{x^{6}\left(b x^{2}+a\right)^{1 / 6}} \mathrm{~d} x
$$

Optimal(type 4, 553 leaves, 9 steps):
$\frac{8 b^{3} x}{27 a^{3}\left(b x^{2}+a\right)^{1 / 6}}-\frac{\left(b x^{2}+a\right)^{5 / 6}}{5 a x^{5}}+\frac{2 b\left(b x^{2}+a\right)^{5 / 6}}{9 a^{2} x^{3}}-\frac{8 b^{2}\left(b x^{2}+a\right)^{5 / 6}}{27 a^{3} x}+\frac{8 b^{3} x}{27 a^{2}\left(\frac{a}{b x^{2}+a}\right)^{2 / 3}\left(b x^{2}+a\right)^{7 / 6}\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)}$

$$
\begin{aligned}
& -\frac{8 b^{2}\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}\right) \text { EllipticF }\left(\frac{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\sqrt{3}}{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3} \sqrt{2} \sqrt{\frac{1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\left(\frac{a}{b x^{2}+a}\right)^{2 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}} 3^{3 / / 4}\right.}{81 a^{2} x\left(\frac{a}{b x^{2}+a}\right)^{2 / 3}\left(b x^{2}+a\right)^{1 / 6} \sqrt{\frac{-1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}}} \\
& +\frac{1}{27 a^{2} x\left(\frac{a}{b x^{2}+a}\right)^{2 / 3}\left(b x^{2}+a\right)^{1 / 6} \sqrt{\frac{-1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}}} 44 b^{2}(1 \\
& \left.\left.\left.-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}\right) \text { EllipticE }\left(\frac{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\sqrt{3}}{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\left(\frac{a}{b x^{2}+a}\right)^{2 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}\left(\frac{\sqrt{6}}{2}\right.}+\frac{\sqrt{2}}{2}\right) 3^{1 / 4}\right)
\end{aligned}
$$

Result(type 8, 58 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{5 / 6}\left(40 b^{2} x^{4}-30 a b x^{2}+27 a^{2}\right)}{135 a^{3} x^{5}}+\int \frac{16 b^{3}}{81 a^{3}\left(b x^{2}+a\right)^{1 / 6}} \mathrm{~d} x
$$

Problem 267: Unable to integrate problem.

$$
\int \frac{1}{\left(b x^{2}+a\right)^{5 / 6}} \mathrm{~d} x
$$

Optimal(type 4, 212 leaves, 3 steps):
$\frac{1}{b x\left(\frac{a}{b x^{2}+a}\right)^{1 / 3} \sqrt{\frac{-1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}}\left(3^{3 / 4}\left(b x^{2}+a\right)^{1 / 6}\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}\right) \operatorname{EllipticF}\left(\frac{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\sqrt{3}}{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}\right.\right.}$

$$
\left.-\mathrm{I} \sqrt{3})\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{\frac{1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\left(\frac{a}{b x^{2}+a}\right)^{2 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}}\right)
$$

Result(type 8, 11 leaves):

$$
\int \frac{1}{\left(b x^{2}+a\right)^{5 / 6}} \mathrm{~d} x
$$

Problem 268: Unable to integrate problem.

$$
\int \frac{1}{x^{2}\left(b x^{2}+a\right)^{5 / 6}} \mathrm{~d} x
$$

Optimal(type 4, 231 leaves, 4 steps):

$$
\begin{aligned}
& -\frac{\left(b x^{2}+a\right)^{1 / 6}}{a x}-\frac{1}{3 a x\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}\left(\frac{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}\right.}\left(2\left(b x^{2}+a\right)^{1 / 6}(1\right. \\
& \left.\left.-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}\right) \text { EllipticF }\left(\frac{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\sqrt{3}}{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right)\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right) / \frac{1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\left(\frac{a}{b x^{2}+a}\right)^{2 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}\right)
\end{aligned}
$$

Result(type 8, 58 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{1 / 6}}{a x}+\frac{\left(\int-\frac{2 b}{3 a\left(\left(b x^{2}+a\right)^{5}\right)^{1 / 6}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right)^{5}\right)^{1 / 6}}{\left(b x^{2}+a\right)^{5 / 6}}
$$

Problem 269: Unable to integrate problem.

$$
\int \frac{1}{x^{4}\left(b x^{2}+a\right)^{7 / 6}} \mathrm{~d} x
$$

Optimal(type 4, 546 leaves, 9 steps):

$$
\begin{aligned}
& \frac{3}{a x^{3}\left(b x^{2}+a\right)^{1 / 6}}-\frac{40 b^{2} x}{9 a^{3}\left(b x^{2}+a\right)^{1 / 6}}-\frac{10\left(b x^{2}+a\right)^{5 / 6}}{3 a^{2} x^{3}}+\frac{40 b\left(b x^{2}+a\right)^{5 / 6}}{9 a^{3} x}-\frac{40 b^{2} x}{9 a^{2}\left(\frac{a}{b x^{2}+a}\right)^{2 / 3}\left(b x^{2}+a\right)^{7 / 6}\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)} \\
& +\frac{40 b\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}\right) \text { EllipticF }\left(\frac{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\sqrt{3}}{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3} \sqrt{2} \sqrt{\frac{1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\left(\frac{a}{b x^{2}+a}\right)^{2 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}} 3^{3 / 4}\right.}{27 a^{2} x\left(\frac{a}{b x^{2}+a}\right)^{2 / 3}\left(b x^{2}+a\right)^{1 / 6} / \sqrt{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}} \\
& -\frac{1}{9 a^{2} x\left(\frac{a}{b x^{2}+a}\right)^{2 / 3}\left(b x^{2}+a\right)^{1 / 6} \sqrt{\frac{-1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}}} \\
& \left.\left.\left.-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}\right) \text { EllipticE }\left(\frac{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\sqrt{3}}{1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) / \frac{1+\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}+\left(\frac{a}{b x^{2}+a}\right)^{2 / 3}}{\left(1-\left(\frac{a}{b x^{2}+a}\right)^{1 / 3}-\sqrt{3}\right)^{2}}\left(\frac{\sqrt{6}}{2}\right)+\frac{\sqrt{2}}{2}\right) 3^{1 / 4}\right)
\end{aligned}
$$

Result(type 8, 66 leaves):

$$
-\frac{\left(b x^{2}+a\right)^{5 / 6}\left(-13 b x^{2}+3 a\right)}{9 a^{3} x^{3}}+\int-\frac{b\left(26 b x^{2}-a\right)}{27 a^{3}\left(x^{2}+\frac{a}{b}\right)\left(b x^{2}+a\right)^{1 / 6}} \mathrm{~d} x
$$

Problem 270: Unable to integrate problem.

$$
\int x^{6}\left(b x^{2}+a\right)^{p} \mathrm{~d} x
$$

Optimal(type 5, 36 leaves, 2 steps):

$$
\frac{x^{7}\left(b x^{2}+a\right)^{1+p} \text { hypergeom }\left(\left[1, \frac{9}{2}+p\right],\left[\frac{9}{2}\right],-\frac{b x^{2}}{a}\right)}{7 a}
$$

Result(type 8, 15 leaves):

$$
\int x^{6}\left(b x^{2}+a\right)^{p} \mathrm{~d} x
$$

Problem 271: Unable to integrate problem.

$$
\int x^{7 / 2}\left(b x^{2}+a\right)^{p} \mathrm{~d} x
$$

Optimal(type 5, 36 leaves, 2 steps):

$$
\frac{2 x^{9 / 2}\left(b x^{2}+a\right)^{1+p} \text { hypergeom }\left(\left[1, \frac{13}{4}+p\right],\left[\frac{13}{4}\right],-\frac{b x^{2}}{a}\right)}{9 a}
$$

Result(type 8, 15 leaves):

$$
\int x^{7 / 2}\left(b x^{2}+a\right)^{p} \mathrm{~d} x
$$

Problem 272: Unable to integrate problem.

$$
\int \sqrt{x}\left(b x^{2}+a\right)^{p} \mathrm{~d} x
$$

Optimal(type 5, 36 leaves, 2 steps):

$$
2 x^{3 / 2}\left(b x^{2}+a\right)^{1+p} \text { hypergeom }\left(\left[1, \frac{7}{4}+p\right],\left[\frac{7}{4}\right],-\frac{b x^{2}}{a}\right)
$$

Result(type 8, 15 leaves):

$$
\int \sqrt{x}\left(b x^{2}+a\right)^{p} \mathrm{~d} x
$$

Problem 273: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{p}}{x^{7 / 2}} d x
$$

Optimal(type 5, 36 leaves, 2 steps):

$$
-\frac{2\left(b x^{2}+a\right)^{1+p} \text { hypergeom }\left(\left[1,-\frac{1}{4}+p\right],\left[-\frac{1}{4}\right],-\frac{b x^{2}}{a}\right)}{5 a x^{5 / 2}}
$$

Result(type 8, 15 leaves):

$$
\int \frac{\left(b x^{2}+a\right)^{p}}{x^{7 / 2}} \mathrm{~d} x
$$

Problem 274: Unable to integrate problem.

$$
\int(c x)^{m}\left(b x^{2}+a\right)^{p} \mathrm{~d} x
$$

Optimal(type 5, 64 leaves, 2 steps):

$$
\frac{(c x)^{1+m}\left(b x^{2}+a\right)^{p} \text { hypergeom }\left(\left[-p, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{c(1+m)\left(1+\frac{b x^{2}}{a}\right)^{p}}
$$

Result(type 8, 17 leaves):

$$
\int(c x)^{m}\left(b x^{2}+a\right)^{p} \mathrm{~d} x
$$

Problem 275: Unable to integrate problem.

$$
\int x^{-8-2 p}\left(b x^{2}+a\right)^{p} \mathrm{~d} x
$$

Optimal(type 5, 49 leaves, 2 steps):

$$
-\frac{x^{-7-2 p}\left(b x^{2}+a\right)^{1+p} \text { hypergeom }\left(\left[-\frac{5}{2}, 1\right],\left[-\frac{5}{2}-p\right],-\frac{b x^{2}}{a}\right)}{a(7+2 p)}
$$

Result(type 8, 19 leaves):

$$
\int x^{-8-2 p}\left(b x^{2}+a\right)^{p} \mathrm{~d} x
$$

Problem 277: Unable to integrate problem.

$$
\int x^{-1-2 p}\left(b x^{2}+a\right)^{p} \mathrm{~d} x
$$

Optimal(type 5, 45 leaves, 2 steps):

$$
-\frac{\left(b x^{2}+a\right)^{1+p} \text { hypergeom }\left([1,1],[1-p],-\frac{b x^{2}}{a}\right)}{2 a p x^{2 p}}
$$

Result(type 8, 19 leaves):

$$
\int x^{-1-2 p}\left(b x^{2}+a\right)^{p} \mathrm{~d} x
$$

Test results for the 96 problems in "1.1.2.3 (a+b $\left.x^{\wedge} 2\right)^{\wedge} p\left(c+d x^{\wedge} 2\right)^{\wedge} q . t x t "$
Problem 8: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(b x^{2}+a\right)^{3}}{\left(d x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 93 leaves, 4 steps):

$$
-\frac{b^{2}(-3 a d+2 b c) x}{d^{3}}+\frac{b^{3} x^{3}}{3 d^{2}}-\frac{(-a d+b c)^{3} x}{2 c d^{3}\left(d x^{2}+c\right)}+\frac{(-a d+b c)^{2}(a d+5 b c) \arctan \left(\frac{x \sqrt{d}}{\sqrt{c}}\right)}{2 c^{3 / 2} d^{7 / 2}}
$$

Result(type 3, 204 leaves):

$$
\begin{aligned}
\frac{b^{3} x^{3}}{3 d^{2}} & +\frac{3 b^{2} a x}{d^{2}}-\frac{2 b^{3} c x}{d^{3}}+\frac{x a^{3}}{2 c\left(d x^{2}+c\right)}-\frac{3 x a^{2} b}{2 d\left(d x^{2}+c\right)}+\frac{3 c x a b^{2}}{2 d^{2}\left(d x^{2}+c\right)}-\frac{c^{2} x b^{3}}{2 d^{3}\left(d x^{2}+c\right)}+\frac{\arctan \left(\frac{x d}{\sqrt{c d}}\right) a^{3}}{2 c \sqrt{c d}}+\frac{3 \arctan \left(\frac{x d}{\sqrt{c d}}\right) a^{2} b}{2 d \sqrt{c d}} \\
& -\frac{9 c \arctan \left(\frac{x d}{\sqrt{c d}}\right) a b^{2}}{2 d^{2} \sqrt{c d}}+\frac{5 c^{2} \arctan \left(\frac{x d}{\sqrt{c d}}\right) b^{3}}{2 d^{3} \sqrt{c d}}
\end{aligned}
$$

Problem 9: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(b x^{2}+a\right)^{3}}{\left(d x^{2}+c\right)^{3}} \mathrm{~d} x
$$

Optimal(type 3, 116 leaves, 5 steps):

$$
\frac{b^{3} x}{d^{3}}-\frac{(-a d+b c)^{3} x}{4 c d^{3}\left(d x^{2}+c\right)^{2}}+\frac{3(-a d+b c)^{2}(a d+3 b c) x}{8 c^{2} d^{3}\left(d x^{2}+c\right)}-\frac{3(-a d+b c)\left(4 b^{2} c^{2}+(a d+b c)^{2}\right) \arctan \left(\frac{x \sqrt{d}}{\sqrt{c}}\right)}{8 c^{5 / 2} d^{7 / 2}}
$$

Result(type 3, 265 leaves):

$$
\begin{aligned}
\frac{b^{3} x}{d^{3}} & +\frac{3 d x^{3} a^{3}}{8\left(d x^{2}+c\right)^{2} c^{2}}+\frac{3 x^{3} a^{2} b}{8\left(d x^{2}+c\right)^{2} c}-\frac{15 x^{3} a b^{2}}{8 d\left(d x^{2}+c\right)^{2}}+\frac{9 c x^{3} b^{3}}{8 d^{2}\left(d x^{2}+c\right)^{2}}+\frac{5 x a^{3}}{8\left(d x^{2}+c\right)^{2} c}-\frac{3 x a^{2} b}{8 d\left(d x^{2}+c\right)^{2}}-\frac{9 c x a b^{2}}{8 d^{2}\left(d x^{2}+c\right)^{2}} \\
& +\frac{7 c^{2} x b^{3}}{8 d^{3}\left(d x^{2}+c\right)^{2}}+\frac{3 \arctan \left(\frac{x d}{\sqrt{c d}}\right) a^{3}}{8 c^{2} \sqrt{c d}}+\frac{3 \arctan \left(\frac{x d}{\sqrt{c d}}\right) a^{2} b}{8 d c \sqrt{c d}}+\frac{9 \arctan \left(\frac{x d}{\sqrt{c d}}\right) a b^{2}}{8 d^{2} \sqrt{c d}}-\frac{15 c \arctan \left(\frac{x d}{\sqrt{c d}}\right) b^{3}}{8 d^{3} \sqrt{c d}}
\end{aligned}
$$

Problem 11: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(d x^{2}+c\right)^{3}}{\left(b x^{2}+a\right)^{2}} d x
$$

Optimal(type 3, 92 leaves, 4 steps):

$$
\frac{d^{2}(-2 a d+3 b c) x}{b^{3}}+\frac{d^{3} x^{3}}{3 b^{2}}+\frac{(-a d+b c)^{3} x}{2 a b^{3}\left(b x^{2}+a\right)}+\frac{(-a d+b c)^{2}(5 a d+b c) \arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2 a^{3 / 2} b^{7 / 2}}
$$

Result(type 3, 204 leaves):

$$
\begin{aligned}
& \frac{d^{3} x^{3}}{3 b^{2}}-\frac{2 d^{3} a x}{b^{3}}+\frac{3 d^{2} c x}{b^{2}}-\frac{x a^{2} d^{3}}{2 b^{3}\left(b x^{2}+a\right)}+\frac{3 x a c d^{2}}{2 b^{2}\left(b x^{2}+a\right)}-\frac{3 x c^{2} d}{2 b\left(b x^{2}+a\right)}+\frac{x c^{3}}{2 a\left(b x^{2}+a\right)}+\frac{5 a^{2} \arctan \left(\frac{b x}{\sqrt{a b}}\right) d^{3}}{2 b^{3} \sqrt{a b}}-\frac{9 a \arctan \left(\frac{b x}{\sqrt{a b}}\right) c d^{2}}{2 b^{2} \sqrt{a b}} \\
& \quad+\frac{3 \arctan \left(\frac{b x}{\sqrt{a b}}\right) c^{2} d}{2 b \sqrt{a b}}+\frac{\arctan \left(\frac{b x}{\sqrt{a b}}\right) c^{3}}{2 a \sqrt{a b}}
\end{aligned}
$$

Problem 12: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(d x^{2}+c\right)^{5}}{\left(b x^{2}+a\right)^{3}} \mathrm{~d} x
$$

Optimal(type 3, 178 leaves, 5 steps):

$$
\begin{aligned}
& \frac{d^{3}\left(6 a^{2} d^{2}-15 a b c d+10 b^{2} c^{2}\right) x}{b^{5}}+\frac{d^{4}(-3 a d+5 b c) x^{3}}{3 b^{4}}+\frac{d^{5} x^{5}}{5 b^{3}}+\frac{(-a d+b c)^{5} x}{4 a b^{5}\left(b x^{2}+a\right)^{2}}+\frac{(-a d+b c)^{4}(17 a d+3 b c) x}{8 a^{2} b^{5}\left(b x^{2}+a\right)} \\
& +\frac{(-a d+b c)^{3}\left(63 a^{2} d^{2}+14 a b c d+3 b^{2} c^{2}\right) \arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{8 a^{5 / 2} b^{11 / 2}}
\end{aligned}
$$

Result(type 3, 483 leaves):
$\frac{d^{5} x^{5}}{5 b^{3}}-\frac{d^{5} x^{3} a}{b^{4}}+\frac{5 d^{4} x^{3} c}{3 b^{3}}+\frac{6 d^{5} a^{2} x}{b^{5}}-\frac{15 d^{4} a c x}{b^{4}}+\frac{10 d^{3} c^{2} x}{b^{3}}+\frac{17 a^{3} x^{3} d^{5}}{8 b^{4}\left(b x^{2}+a\right)^{2}}-\frac{65 a^{2} x^{3} c d^{4}}{8 b^{3}\left(b x^{2}+a\right)^{2}}+\frac{45 a x^{3} c^{2} d^{3}}{4 b^{2}\left(b x^{2}+a\right)^{2}}-\frac{25 x^{3} c^{3} d^{2}}{4 b\left(b x^{2}+a\right)^{2}}$
$+\frac{5 x^{3} c^{4} d}{8\left(b x^{2}+a\right)^{2} a}+\frac{3 b x^{3} c^{5}}{8\left(b x^{2}+a\right)^{2} a^{2}}+\frac{15 a^{4} x d^{5}}{8 b^{5}\left(b x^{2}+a\right)^{2}}-\frac{55 a^{3} x c d^{4}}{8 b^{4}\left(b x^{2}+a\right)^{2}}+\frac{35 a^{2} x c^{2} d^{3}}{4 b^{3}\left(b x^{2}+a\right)^{2}}-\frac{15 a x c^{3} d^{2}}{4 b^{2}\left(b x^{2}+a\right)^{2}}-\frac{5 x c^{4} d}{8 b\left(b x^{2}+a\right)^{2}}$
$+\frac{5 x c^{5}}{8\left(b x^{2}+a\right)^{2} a}-\frac{63 a^{3} \arctan \left(\frac{b x}{\sqrt{a b}}\right) d^{5}}{8 b^{5} \sqrt{a b}}+\frac{175 a^{2} \arctan \left(\frac{b x}{\sqrt{a b}}\right) c d^{4}}{8 b^{4} \sqrt{a b}}-\frac{75 a \arctan \left(\frac{b x}{\sqrt{a b}}\right) c^{2} d^{3}}{4 b^{3} \sqrt{a b}}+\frac{15 \arctan \left(\frac{b x}{\sqrt{a b}}\right) c^{3} d^{2}}{4 b^{2} \sqrt{a b}}$
$+\frac{5 \arctan \left(\frac{b x}{\sqrt{a b}}\right) c^{4} d}{8 b a \sqrt{a b}}+\frac{3 \arctan \left(\frac{b x}{\sqrt{a b}}\right) c^{5}}{8 a^{2} \sqrt{a b}}$

Problem 17: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{b x^{2}+a}}{d x^{2}+c} d x
$$

Optimal(type 3, 66 leaves, 5 steps):

$$
\frac{\operatorname{arctanh}\left(\frac{x \sqrt{b}}{\sqrt{b x^{2}+a}}\right) \sqrt{b}}{d}-\frac{\operatorname{arctanh}\left(\frac{x \sqrt{-a d+b c}}{\sqrt{c} \sqrt{b x^{2}+a}}\right) \sqrt{-a d+b c}}{d \sqrt{c}}
$$

Result(type 3, 931 leaves):

$$
-\frac{\sqrt{\left(x+\frac{\sqrt{-c d}}{d}\right)^{2} b-\frac{2 b \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}}{2 \sqrt{-c d}}
$$

$$
+\frac{\sqrt{b} \ln \left(\frac{-\frac{b \sqrt{-c d}}{d}+b\left(x+\frac{\sqrt{-c d}}{d}\right)}{\sqrt{b}}+\sqrt{\left.\left(x+\frac{\sqrt{-c d}}{d}\right)^{2} b-\frac{2 b \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}\right)}\right.}{2 d}
$$

$$
\begin{aligned}
& \sqrt{\left(x-\frac{\sqrt{-c d}}{d}\right)^{2} b+\frac{2 b \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}} \\
& 2 \sqrt{-c d} \\
& +\frac{\sqrt{b} \ln \left(\frac{\frac{b \sqrt{-c d}}{d}+b\left(x-\frac{\sqrt{-c d}}{d}\right)}{\sqrt{b}}+\sqrt{\left(x-\frac{\sqrt{-c d}}{d}\right)^{2} b+\frac{2 b \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}\right)}{2 d} \\
& \ln \left(\frac{\frac{2(a d-b c)}{d}+\frac{2 b \sqrt{-c d}}{}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{\frac{a d-b c}{d}} \sqrt{x-\frac{\sqrt{-c d}}{d}}\right) \\
& 2 \sqrt{-c d} \sqrt{\frac{a d-b c}{d}} \\
& +\frac{\ln \left(\frac{2(a d-b c)}{d}+\frac{2 b \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{\frac{a d-b c}{d}} \sqrt{x-\frac{\sqrt{-c d}}{d}}\right)}{2 \sqrt{\left.d-\frac{\sqrt{-c d}}{d}\right)^{2}} \sqrt{\left.d+\frac{2 b \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right.}{d}\right)}}
\end{aligned}
$$



Problem 19: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(b x^{2}+a\right)^{3 / 2}}{\left(d x^{2}+c\right)^{3}} \mathrm{~d} x
$$

Optimal(type 3, 93 leaves, 4 steps):

$$
\frac{x\left(b x^{2}+a\right)^{3 / 2}}{4 c\left(d x^{2}+c\right)^{2}}+\frac{3 a^{2} \operatorname{arctanh}\left(\frac{x \sqrt{-a d+b c}}{\sqrt{c} \sqrt{b x^{2}+a}}\right)}{8 c^{5 / 2} \sqrt{-a d+b c}}+\frac{3 a x \sqrt{b x^{2}+a}}{8 c^{2}\left(d x^{2}+c\right)}
$$

Result (type ?, 9058 leaves): Display of huge result suppressed!
Problem 22: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(b x^{2}+a\right)^{5 / 2}}{d x^{2}+c} \mathrm{~d} x
$$

Optimal (type 3, 131 leaves, 7 steps):
$\frac{b x\left(b x^{2}+a\right)^{3 / 2}}{4 d}+\frac{\left(15 a^{2} d^{2}-20 a b c d+8 b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{x \sqrt{b}}{\sqrt{b x^{2}+a}}\right) \sqrt{b}}{8 d^{3}}-\frac{(-a d+b c)^{5 / 2} \operatorname{arctanh}\left(\frac{x \sqrt{-a d+b c}}{\sqrt{c} \sqrt{b x^{2}+a}}\right)}{d^{3} \sqrt{c}}$

$$
-\frac{b(-7 a d+4 b c) x \sqrt{b x^{2}+a}}{8 d^{2}}
$$

Result(type ?, 3052 leaves): Display of huge result suppressed!

Problem 23: Humongous result has more than 20000 leaves.

$$
\int \frac{\left(b x^{2}+a\right)^{5 / 2}}{\left(d x^{2}+c\right)^{5}} d x
$$

Optimal(type 3, 221 leaves, 6 steps):

$$
\begin{aligned}
& -\frac{d x\left(b x^{2}+a\right)^{7 / 2}}{8 c(-a d+b c)\left(d x^{2}+c\right)^{4}}+\frac{(-7 a d+8 b c) x\left(b x^{2}+a\right)^{5 / 2}}{48 c^{2}(-a d+b c)\left(d x^{2}+c\right)^{3}}+\frac{5 a(-7 a d+8 b c) x\left(b x^{2}+a\right)^{3 / 2}}{192 c^{3}(-a d+b c)\left(d x^{2}+c\right)^{2}}+\frac{5 a^{3}(-7 a d+8 b c) \operatorname{arctanh}\left(\frac{x \sqrt{-a d+b c}}{\left.\sqrt{c} \sqrt{b x^{2}+a}\right)}\right.}{\quad+\frac{5 a^{2}(-7 a d+8 b c) x \sqrt{b x^{2}+a}}{128 c^{4}(-a d+b c)\left(d x^{2}+c\right)}}
\end{aligned}
$$

Result(type ?, 28624 leaves): Display of huge result suppressed!
Problem 24: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{\sqrt{b x^{2}+a}\left(d x^{2}+c\right)^{3}} \mathrm{~d} x
$$

Optimal(type 3, 143 leaves, 5 steps):

$$
\frac{\left(3 a^{2} d^{2}-8 a b c d+8 b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{x \sqrt{-a d+b c}}{\sqrt{c} \sqrt{b x^{2}+a}}\right)}{8 c^{5 / 2}(-a d+b c)^{5 / 2}}-\frac{d x \sqrt{b x^{2}+a}}{4 c(-a d+b c)\left(d x^{2}+c\right)^{2}}-\frac{3 d(-a d+2 b c) x \sqrt{b x^{2}+a}}{8 c^{2}(-a d+b c)^{2}\left(d x^{2}+c\right)}
$$

Result (type 3, 1814 leaves):

$$
\frac{3 \sqrt{\left(x-\frac{\sqrt{-c d}}{d}\right)^{2} b+\frac{2 b \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}}{16 c^{2}(a d-b c)\left(x-\frac{\sqrt{-c d}}{d}\right)}
$$

$$
-3 b \sqrt{-c d} \ln \left(\frac{\left.\frac{2(a d-b c)}{d}+\frac{2 b \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{\frac{a d-b c}{d}} \sqrt{\left(x-\frac{\sqrt{-c d}}{d}\right)^{2} b+\frac{2 b \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}\right)}{d x}\right)
$$

$$
16 c^{2} d(a d-b c) \sqrt{\frac{a d-b c}{d}}
$$

$$
\begin{aligned}
& +\frac{3 \sqrt{\left(x+\frac{\sqrt{-c d}}{d}\right)^{2} b-\frac{2 b \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}}{16 c^{2}(a d-b c)\left(x+\frac{\sqrt{-c d}}{d}\right)} \\
& 3 b \sqrt{-c d} \ln \left(\frac{\left.\frac{2(a d-b c)}{d}-\frac{2 b \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{\frac{a d-b c}{d}} \sqrt{\left(x+\frac{\sqrt{-c d}}{d}\right)^{2} b-\frac{2 b \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}\right)}{x+\frac{\sqrt{-c d}}{d}}\right. \\
& +\overline{16 c^{2} d(a d-b c) \sqrt{\frac{a d-b c}{d}}} \\
& 3 \ln \left(\frac{\left.\frac{2(a d-b c)}{d}+\frac{2 b \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{\frac{a d-b c}{d}} \sqrt{\left(x-\frac{\sqrt{-c d}}{d}\right)^{2} b+\frac{2 b \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}\right)}{d-\frac{\sqrt{-c d}}{d}}\right) \\
& 16 \sqrt{-c d} c^{2} \sqrt{\frac{a d-b c}{d}} \\
& 3 \ln \left(\frac{\left.\frac{2(a d-b c)}{d}-\frac{2 b \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{\frac{a d-b c}{d}} \sqrt{\left(x+\frac{\sqrt{-c d}}{d}\right)^{2} b-\frac{2 b \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}\right)}{16 \sqrt{-c d} c^{2} \sqrt{\frac{a d-b c}{d}}}\right) \\
& +\frac{\sqrt{\left(x-\frac{\sqrt{-c d}}{d}\right)^{2} b+\frac{2 b \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}}{16 \sqrt{-c d} c(a d-b c)\left(x-\frac{\sqrt{-c d}}{d}\right)^{2}}-\frac{3 b \sqrt{\left(x-\frac{\sqrt{-c d}}{d}\right)^{2} b+\frac{2 b \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}}{16 c(a d-b c)^{2}\left(x-\frac{\sqrt{-c d}}{d}\right)}
\end{aligned}
$$

$$
\begin{aligned}
& -3 b^{2} \ln \left(\frac{\frac{2(a d-b c)}{d}+\frac{2 b \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{\frac{a d-b c}{d}} \sqrt{\left(x-\frac{\sqrt{-c d}}{d}\right)^{2} b+\frac{2 b \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}}{x-\frac{\sqrt{-c d}}{d}}\right) \\
& \left.\begin{array}{c}
16 \sqrt{-c d}(a d-b c)^{2} \sqrt{\frac{a d-b c}{d}} \\
b \ln \left(\frac{2(a d-b c)}{d}+\frac{2 b \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{\frac{a d-b c}{d} \sqrt{\left(x-\frac{\sqrt{-c d}}{d}\right)^{2} b+\frac{2 b \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}} \underset{x-\frac{\sqrt{-c d}}{d}}{d}\right.
\end{array}\right) \\
& 16 \sqrt{-c d} c(a d-b c) \sqrt{\frac{a d-b c}{d}} \\
& -\frac{\sqrt{\left(x+\frac{\sqrt{-c d}}{d}\right)^{2} b-\frac{2 b \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}}{16 \sqrt{-c d} c(a d-b c)\left(x+\frac{\sqrt{-c d}}{d}\right)^{2}}-\frac{3 b \sqrt{\left(x+\frac{\sqrt{-c d}}{d}\right)^{2} b-\frac{2 b \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}}{16 c(a d-b c)^{2}\left(x+\frac{\sqrt{-c d}}{d}\right)} \\
& 3 b^{2} \ln \left(\frac{\frac{2(a d-b c)}{d}-\frac{2 b \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{\frac{a d-b c}{d}} \sqrt{\left(x+\frac{\sqrt{-c d}}{d}\right)^{2} b-\frac{2 b \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}}{x+\frac{\sqrt{-c d}}{d}}\right) \\
& +\longrightarrow 16 \sqrt{-c d}(a d-b c)^{2} \sqrt{\frac{a d-b c}{d}} \\
& b \ln \left(\frac{\frac{2(a d-b c)}{d}-\frac{2 b \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{\frac{a d-b c}{d}} \sqrt{\left(x+\frac{\sqrt{-c d}}{d}\right)^{2} b-\frac{2 b \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+\frac{a d-b c}{d}}}{x+\frac{\sqrt{-c d}}{d}}\right) \\
& 16 \sqrt{-c d} c(a d-b c) \sqrt{\frac{a d-b c}{d}}
\end{aligned}
$$

[^0]$$
\int \frac{1}{\left(b x^{2}+a\right)\left(d x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 67 leaves, 3 steps):

$$
\frac{b \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{(-a d+b c)^{3 / 2} \sqrt{a}}-\frac{d x}{c(-a d+b c) \sqrt{d x^{2}+c}}
$$

Result(type 3, 627 leaves):


$$
+\frac{b}{2 \sqrt{-a b}(a d-b c) \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}
$$

$$
+\frac{x d}{2(a d-b c) c \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}
$$

$$
\left.-\frac{b \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right.}{2 \sqrt{-a b}(a d-b c) \sqrt{-\frac{a d-b c}{b}}}\right)
$$

Problem 30: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{\left(b x^{2}+a\right)^{2} \sqrt{d x^{2}+c}} \mathrm{~d} x
$$

Optimal(type 3, 84 leaves, 3 steps):

$$
\frac{(-2 a d+b c) \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{2 a^{3 / 2}(-a d+b c)^{3 / 2}}+\frac{b x \sqrt{d x^{2}+c}}{2 a(-a d+b c)\left(b x^{2}+a\right)}
$$

Result(type 3, 822 leaves):

$$
-\frac{\sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{4 a(a d-b c)\left(x-\frac{\sqrt{-a b}}{b}\right)}
$$

$$
d \sqrt{-a b} \ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right)
$$

$$
-\frac{\sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{}
$$

$$
4 a(a d-b c)\left(x+\frac{\sqrt{-a b}}{b}\right)
$$

$$
\begin{aligned}
& d \sqrt{-a b} \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{x+\frac{\sqrt{-a b}}{b}}}{\left.x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right) \\
& 4 a b(a d-b c) \sqrt{-\frac{a d-b c}{b}} \\
& \ln \left(\frac{\left.-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{x-\frac{\sqrt{-a b}}{b}}-\frac{a d-b c}{b}}\right)}{}\right. \\
& 4 a \sqrt{-a b} \sqrt{-\frac{a d-b c}{b}} \\
& +\frac{\ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right)}{4 a \sqrt{-a b} \sqrt{-\frac{a d-b c}{b}}}
\end{aligned}
$$

Problem 31: Unable to integrate problem.

$$
\int \frac{\left(-b x^{2}+a\right)^{2 / 3}}{\left(b x^{2}+3 a\right)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 464 leaves, 6 steps):

$$
\begin{aligned}
& \frac{x\left(-b x^{2}+a\right)^{2 / 3}}{6 a\left(b x^{2}+3 a\right)}-\frac{x}{6 a\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)} \\
& \quad+\frac{1}{18 a^{2 / 3} b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right) \text { EllipticF }\left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left.-\mathrm{I} \sqrt{3}) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}} 3^{3 / 4} \sqrt{2}\right)-\frac{1}{12 a^{2 / 3} b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(\left(a^{1 / 3}\right.\right. \\
& \left.-\left(-b x^{2}+a\right)^{1 / 3}\right) \text { EllipticE }\left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}\right. \\
& \left.\left.+\frac{\sqrt{2}}{2}\right) 3^{1 / 4}\right)
\end{aligned}
$$

Result(type 8, 24 leaves):

$$
\int \frac{\left(-b x^{2}+a\right)^{2 / 3}}{\left(b x^{2}+3 a\right)^{2}} \mathrm{~d} x
$$

Problem 32: Unable to integrate problem.

$$
\int\left(-b x^{2}+a\right)^{5 / 3}\left(b x^{2}+3 a\right)^{2} \mathrm{~d} x
$$

Optimal(type 4, 505 leaves, 8 steps):
$\frac{28512 a^{3} x\left(-b x^{2}+a\right)^{2 / 3}}{8645}+\frac{14256 a^{2} x\left(-b x^{2}+a\right)^{5 / 3}}{6175}-\frac{306 a x\left(-b x^{2}+a\right)^{8 / 3}}{475}-\frac{3 x\left(-b x^{2}+a\right)^{8 / 3}\left(b x^{2}+3 a\right)}{25}$

$$
-\frac{114048 a^{4} x}{8645\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)}+\frac{1}{8645 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(3 8 0 1 6 3 ^ { 3 / 4 } a ^ { 1 3 / 3 } \left(a^{1 / 3}-(\right.\right.
$$

$$
\left.\left.\left.-b x^{2}+a\right)^{1 / 3}\right) \text { EllipticF }\left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{2} \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\right)
$$

$$
-\frac{1}{8645 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(5 7 0 2 4 3 ^ { 1 / 4 } a ^ { 1 3 / 3 } \left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 /}\right.\right.
$$

$$
\left.\left.{ }^{3}\right) \text { EllipticE }\left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\right)
$$

Result(type 8, 63 leaves):

$$
\frac{3 x\left(-1729 b^{3} x^{6}-11011 a b^{2} x^{4}-6055 a^{2} b x^{2}+66315 a^{3}\right)\left(-b x^{2}+a\right)^{2 / 3}}{43225}+\int \frac{38016 a^{4}}{8645\left(-b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 33: Unable to integrate problem.

$$
\int\left(-b x^{2}+a\right)^{5 / 3}\left(b x^{2}+3 a\right) \mathrm{d} x
$$

Optimal(type 4, 480 leaves, 7 steps):

$$
\begin{aligned}
& \frac{1800 a^{2} x\left(-b x^{2}+a\right)^{2 / 3}}{1729}+\frac{180 a x\left(-b x^{2}+a\right)^{5 / 3}}{247}-\frac{3 x\left(-b x^{2}+a\right)^{8 / 3}}{19}-\frac{1}{1729\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)} \\
& +\frac{1}{1729 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(2 4 0 0 3 ^ { 3 / 4 } a ^ { 1 0 / 3 } \left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 /}\right.\right.
\end{aligned}
$$

$$
\left.\left.{ }^{3}\right) \text { EllipticF }\left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{2} \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\right)
$$

$$
-\frac{1}{1729 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(36003^{1 / 4} a^{10 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 /}\right) .\right.}
$$

$$
\left.\left.{ }^{3}\right) \text { EllipticE }\left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\right)
$$

Result(type 8, 52 leaves):

$$
\frac{3 x\left(-91 b^{2} x^{4}-238 a b x^{2}+929 a^{2}\right)\left(-b x^{2}+a\right)^{2 / 3}}{1729}+\int \frac{2400 a^{3}}{1729\left(-b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 34: Unable to integrate problem.

$$
\int \frac{\left(-b x^{2}+a\right)^{5 / 3}}{b x^{2}+3 a} \mathrm{~d} x
$$

Optimal(type 4, 583 leaves, 7 steps):

$$
\begin{aligned}
-\frac{3 x\left(-b x^{2}+a\right)^{2 / 3}}{7}+\frac{96 a x}{7\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)}+\frac{42^{1 / 3} a^{7 / 6} \operatorname{arctanh}\left(\frac{x \sqrt{b}}{a^{1 / 6}\left(a^{1 / 3}+2^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}\right)}\right)}{\sqrt{b}} \\
-\frac{42^{1 / 3} a^{7 / 6} \operatorname{arctanh}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{3 \sqrt{b}}+\frac{42^{1 / 3} a^{7 / 6} \arctan \left(\frac{a^{1 / 6}\left(a^{1 / 3}-2^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}\right) \sqrt{3}}{x \sqrt{b}}\right) \sqrt{3}}{3 \sqrt{b}}+\frac{42^{1 / 3} a^{7 / 6} \arctan \left(\frac{\sqrt{3} \sqrt{a}}{x \sqrt{b}}\right) \sqrt{3}}{3 \sqrt{b}}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{7 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(3 2 3 ^ { 3 / 4 } a ^ { 4 / 3 } ( a ^ { 1 / 3 } - ( - b x ^ { 2 } + a ) ^ { 1 / 3 } ) \text { EllipticF } \left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}\right.\right. \\
& \left.-\mathrm{I} \sqrt{3}) \sqrt{2} \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\right)+\frac{1}{7 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(483^{1 / 4} a^{4 / 3}\left(a^{1 / 3}\right)\right.} \\
& \left.\left.-\left(-b x^{2}+a\right)^{1 / 3}\right) \text { EllipticE }\left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\right)
\end{aligned}
$$

Result(type 8, 54 leaves):

$$
-\frac{3 x\left(-b x^{2}+a\right)^{2 / 3}}{7}+\int-\frac{16 a\left(2 b x^{2}-a\right)}{7 b\left(x^{2}+\frac{3 a}{b}\right)\left(-b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 35: Unable to integrate problem.

$$
\int \frac{\left(-b x^{2}+a\right)^{5 / 3}}{\left(b x^{2}+3 a\right)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 593 leaves, 7 steps):

$$
\begin{aligned}
& \frac{2 x\left(-b x^{2}+a\right)^{2 / 3}}{3\left(b x^{2}+3 a\right)}-\frac{11 x}{3\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)}-\frac{2^{1 / 3} a^{1 / 6} \operatorname{arctanh}\left(\frac{x \sqrt{b}}{a^{1 / 6}\left(a^{1 / 3}+2^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}\right)}\right)}{\sqrt{b}} \\
& +\frac{2^{1 / 3} a^{1 / 6} \operatorname{arctanh}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{3 \sqrt{b}}-\frac{2^{1 / 3} a^{1 / 6} \arctan \left(\frac{a^{1 / 6}\left(a^{1 / 3}-2^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}\right) \sqrt{3}}{x \sqrt{b}}\right) \sqrt{3}}{3 \sqrt{b}}-\frac{2^{1 / 3} a^{1 / 6} \arctan \left(\frac{\sqrt{3} \sqrt{a}}{x \sqrt{b}}\right) \sqrt{3}}{3 \sqrt{b}} \\
& +\frac{1}{9 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(1 1 a ^ { 1 / 3 } ( a ^ { 1 / 3 } - ( - b x ^ { 2 } + a ) ^ { 1 / 3 } ) \text { EllipticF } \left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}\right.\right. \\
& \left.-\mathrm{I} \sqrt{3}) \sqrt{2} \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}} 3^{3 / 4}\right)-\frac{1}{6 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(1 1 a ^ { 1 / 3 } \left(a^{1 / 3}\right.\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left.-\left(-b x^{2}+a\right)^{1 / 3}\right) \text { EllipticE }\left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}\right. \\
& \left.\left.+\frac{\sqrt{2}}{2}\right) 3^{1 / 4}\right)
\end{aligned}
$$

Result(type 8, 24 leaves):

$$
\int \frac{\left(-b x^{2}+a\right)^{5 / 3}}{\left(b x^{2}+3 a\right)^{2}} \mathrm{~d} x
$$

Problem 36: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+3 a\right)^{4}}{\left(-b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Optimal(type 4, 527 leaves, 8 steps):
$-\frac{1552608 a^{3} x\left(-b x^{2}+a\right)^{2 / 3}}{43225}-\frac{36288 a^{2} x\left(-b x^{2}+a\right)^{2 / 3}\left(b x^{2}+3 a\right)}{6175}-\frac{18 a x\left(-b x^{2}+a\right)^{2 / 3}\left(b x^{2}+3 a\right)^{2}}{19}-\frac{3 x\left(-b x^{2}+a\right)^{2 / 3}\left(b x^{2}+3 a\right)^{3}}{25}$

$$
-\frac{3794688 a^{4} x}{8645\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)}+\frac{1}{8645 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(1 2 6 4 8 9 6 3 ^ { 3 / 4 } a ^ { 1 3 / 3 } \left(a^{1 / 3}-(\right.\right.
$$

$$
\left.\left.\left.-b x^{2}+a\right)^{1 / 3}\right) \text { EllipticF }\left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{2} \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\right)
$$

$$
-\frac{1}{8645 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(18973443^{1 / 4} a^{13 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 /}\right)\right.}(
$$

$$
\left.\left.{ }^{3}\right) \text { EllipticE }\left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\right)
$$

Result(type 8, 63 leaves):

$$
-\frac{3 x\left(1729 b^{3} x^{6}+29211 a b^{2} x^{4}+213255 a^{2} b x^{2}+941085 a^{3}\right)\left(-b x^{2}+a\right)^{2 / 3}}{43225}+\int \frac{1264896 a^{4}}{8645\left(-b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 37: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+3 a\right)^{2}}{\left(-b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Optimal(type 4, 473 leaves, 6 steps):

$$
-\frac{198 a x\left(-b x^{2}+a\right)^{2 / 3}}{91}-\frac{3 x\left(-b x^{2}+a\right)^{2 / 3}\left(b x^{2}+3 a\right)}{13}-\frac{3240 a^{2} x}{91\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)}
$$

$$
\begin{aligned}
& +\frac{1}{91 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(1 0 8 0 3 ^ { 3 / 4 } a ^ { 7 / 3 } ( a ^ { 1 / 3 } - ( - b x ^ { 2 } + a ) ^ { 1 / 3 } ) \operatorname { E l l i p t i c F } \left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}\right.\right.} \begin{array}{l}
\left.-\mathrm{I} \sqrt{3}) \sqrt{2} \sqrt{\left.\left.\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3}\right.}\right)\right)^{2}}\right) \\
-\frac{1}{91 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(1 6 2 0 3 ^ { 1 / 4 } a ^ { 7 / 3 } ( a ^ { 1 / 3 } - ( - b x ^ { 2 } + a ) ^ { 1 / 3 } ) \operatorname { E l l i p t i c E } \left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}\right.\right. \\
\left.-\mathrm{I} \sqrt{3}) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\right)
\end{array}
\end{aligned}
$$

Result(type 8, 41 leaves):

$$
-\frac{3 x\left(7 b x^{2}+87 a\right)\left(-b x^{2}+a\right)^{2 / 3}}{91}+\int \frac{1080 a^{2}}{91\left(-b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 38: Unable to integrate problem.

$$
\int \frac{1}{\left(-b x^{2}+a\right)^{1 / 3}\left(b x^{2}+3 a\right)} \mathrm{d} x
$$

Optimal(type 3, 136 leaves, 1 step):

$$
\frac{\operatorname{arctanh}\left(\frac{x \sqrt{b}}{a^{1 / 6}\left(a^{1 / 3}+2^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}\right)}\right) 2^{1 / 3}}{4 a^{5 / 6} \sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right) 2^{1 / 3}}{12 a^{5 / 6} \sqrt{b}}+\frac{\arctan \left(\frac{a^{1 / 6}\left(a^{1 / 3}-2^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}\right) \sqrt{3}}{x \sqrt{b}}\right) 2^{1 / 3} \sqrt{3}}{12 a^{5 / 6} \sqrt{b}}
$$

$$
+\frac{\arctan \left(\frac{\sqrt{3} \sqrt{a}}{x \sqrt{b}}\right) 2^{1 / 3} \sqrt{3}}{12 a^{5 / 6} \sqrt{b}}
$$

Result(type 8, 24 leaves):

$$
\int \frac{1}{\left(-b x^{2}+a\right)^{1 / 3}\left(b x^{2}+3 a\right)} \mathrm{d} x
$$

Problem 39: Unable to integrate problem.

$$
\int \frac{b x^{2}+3 a}{\left(-b x^{2}+a\right)^{4 / 3}} \mathrm{~d} x
$$

Optimal(type 4, 447 leaves, 5 steps):

$$
\begin{aligned}
& \frac{6 x}{\left(-b x^{2}+a\right)^{1 / 3}}+\frac{9 x}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}-\frac{1}{b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(3 a ^ { 1 / 3 } \left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 /}\right.\right. \\
& \text { 3) EllipticF } \left.\left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{2} \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}} 3^{3 / 4}\right) \\
& +\frac{1}{2 b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(9 a ^ { 1 / 3 } ( a ^ { 1 / 3 } - ( - b x ^ { 2 } + a ) ^ { 1 / 3 } ) \text { EllipticE } \left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}\right.\right.} \\
& \left.-\mathrm{I} \sqrt{3}) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right) 3^{1 / 4}\right)
\end{aligned}
$$

Result(type 8, 22 leaves):

$$
\int \frac{b x^{2}+3 a}{\left(-b x^{2}+a\right)^{4 / 3}} \mathrm{~d} x
$$

Problem 40: Unable to integrate problem.

$$
\int \frac{1}{\left(-b x^{2}+a\right)^{4 / 3}\left(b x^{2}+3 a\right)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 615 leaves, 8 steps):

$$
\begin{aligned}
& \frac{x}{12 a^{3}\left(-b x^{2}+a\right)^{1 / 3}}+\frac{x}{24 a^{2}\left(-b x^{2}+a\right)^{1 / 3}\left(b x^{2}+3 a\right)}+\frac{x}{12 a^{3}\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)} \\
& +\frac{\operatorname{arctanh}\left(\frac{x \sqrt{b}}{a^{1 / 6}\left(a^{1 / 3}+2^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}\right)}\right) 2^{1 / 3}}{32 a^{17 / 6} \sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right) 2^{1 / 3}}{96 a^{17 / 6} \sqrt{b}}+\frac{\arctan \left(\frac{a^{1 / 6}\left(a^{1 / 3}-2^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}\right) \sqrt{3}}{x \sqrt{b}}\right) 2^{1 / 3} \sqrt{3}}{96 a^{17 / 6} \sqrt{b}}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{\arctan \left(\frac{\sqrt{3} \sqrt{a}}{x \sqrt{b}}\right) 2^{1 / 3} \sqrt{3}}{96 a^{17 / 6} \sqrt{b}} \\
& -\frac{1}{36 a^{8 / 3} b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(( a ^ { 1 / 3 } - ( - b x ^ { 2 } + a ) ^ { 1 / 3 } ) \operatorname { E l l i p t i c F } \left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}\right.\right. \\
& \left.-\mathrm{I} \sqrt{3}) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}} 3^{3 / 4} \sqrt{2}\right)+\frac{1}{24 a^{8 / 3} b x \sqrt{-\frac{a^{1 / 3}\left(a^{1 / 3}-\left(-b x^{2}+a\right)^{1 / 3}\right)}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}}\left(\left(a^{1 / 3}\right.\right. \\
& \left.-\left(-b x^{2}+a\right)^{1 / 3}\right) \operatorname{EllipticE}\left(\frac{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1+\sqrt{3})}{-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{a^{2 / 3}+a^{1 / 3}\left(-b x^{2}+a\right)^{1 / 3}+\left(-b x^{2}+a\right)^{2 / 3}}{\left(-\left(-b x^{2}+a\right)^{1 / 3}+a^{1 / 3}(1-\sqrt{3})\right)^{2}}}\left(\frac{\sqrt{6}}{2}\right. \\
& \left.\left.+\frac{\sqrt{2}}{2}\right) 3^{1 / 4}\right)
\end{aligned}
$$

Result(type 8, 24 leaves):

$$
\int \frac{1}{\left(-b x^{2}+a\right)^{4 / 3}\left(b x^{2}+3 a\right)^{2}} \mathrm{~d} x
$$

Problem 41: Unable to integrate problem.

$$
\int \frac{1}{\left(-b x^{2}+3 a\right)\left(b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Optimal (type 3, 134 leaves, 1 step):

$$
\begin{aligned}
& \frac{\arctan \left(\frac{x \sqrt{b}}{a^{1 / 6}\left(a^{1 / 3}+2^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}\right)}\right) 2^{1 / 3}}{4 a^{5 / 6} \sqrt{b}}-\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right) 2^{1 / 3}}{12 a^{5 / 6} \sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{a^{1 / 6}\left(a^{1 / 3}-2^{1 / 3}\left(b x^{2}+a\right)^{1 / 3}\right) \sqrt{3}}{x \sqrt{b}}\right) 2^{1 / 3} \sqrt{3}}{12 a^{5 / 6} \sqrt{b}} \\
& \quad-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3} \sqrt{a}}{x \sqrt{b}}\right) 2^{1 / 3} \sqrt{3}}{12 a^{5} / 6 \sqrt{b}}
\end{aligned}
$$

Result(type 8, 24 leaves):

$$
\int \frac{1}{\left(-b x^{2}+3 a\right)\left(b x^{2}+a\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 42: Unable to integrate problem.

$$
\int \frac{1}{\left(-x^{2}+3\right)\left(x^{2}+1\right)^{1 / 3}} d x
$$

Optimal(type 3, 77 leaves, 1 step):

$$
-\frac{\arctan (x) 2^{1 / 3}}{12}+\frac{\arctan \left(\frac{x}{1+2^{1 / 3}\left(x^{2}+1\right)^{1 / 3}}\right) 2^{1 / 3}}{4}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}}{x}\right) 2^{1 / 3} \sqrt{3}}{12}-\frac{\operatorname{arctanh}\left(\frac{\left(1-2^{1 / 3}\left(x^{2}+1\right)^{1 / 3}\right) \sqrt{3}}{x}\right) 2^{1 / 3} \sqrt{3}}{12}
$$

Result(type 8, 19 leaves):

$$
\int \frac{1}{\left(-x^{2}+3\right)\left(x^{2}+1\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 43: Unable to integrate problem.

$$
\int \frac{3-x}{\left(-x^{2}+1\right)^{1 / 3}\left(x^{2}+3\right)} \mathrm{d} x
$$

Optimal(type 3, 70 leaves, 1 step):

$$
-\frac{\ln \left(x^{2}+3\right) 2^{1 / 3}}{4}+\frac{3 \ln \left(2^{1 / 3}(1-x)^{1 / 3}+(x+1)^{2 / 3}\right) 2^{1 / 3}}{4}+\frac{\arctan \left(-\frac{\sqrt{3}}{3}+\frac{2^{2 / 3}(x+1)^{2 / 3} \sqrt{3}}{3(1-x)^{1 / 3}}\right) \sqrt{3} 2^{1 / 3}}{2}
$$

Result(type 8, 24 leaves):

$$
\int \frac{3-x}{\left(-x^{2}+1\right)^{1 / 3}\left(x^{2}+3\right)} \mathrm{d} x
$$

Problem 44: Unable to integrate problem.

$$
\int \frac{1}{\left(b x^{2}-a\right)^{1 / 3}\left(-\frac{9 a d}{b}+d x^{2}\right)} \mathrm{d} x
$$

Optimal(type 3, 105 leaves, 1 step):

$$
-\frac{\operatorname{arctanh}\left(\frac{\left(a^{1 / 3}+\left(b x^{2}-a\right)^{1 / 3}\right)^{2}}{3 a^{1 / 6} x \sqrt{b}}\right) \sqrt{b}}{12 a^{5 / 6} d}+\frac{\operatorname{arctanh}\left(\frac{x \sqrt{b}}{3 \sqrt{a}}\right) \sqrt{b}}{12 a^{5 / 6} d}+\frac{\arctan \left(\frac{a^{1 / 6}\left(a^{1 / 3}+\left(b x^{2}-a\right)^{1 / 3}\right) \sqrt{3}}{x \sqrt{b}}\right) \sqrt{b} \sqrt{3}}{12 a^{5 / 6} d}
$$

Result(type 8, 29 leaves):

$$
\int \frac{1}{\left(b x^{2}-a\right)^{1 / 3}\left(-\frac{9 a d}{b}+d x^{2}\right)} \mathrm{d} x
$$

Problem 45: Unable to integrate problem.

$$
\int \frac{1}{\left(b x^{2}-2\right)^{1 / 3}\left(-\frac{18 d}{b}+d x^{2}\right)} \mathrm{d} x
$$

Optimal(type 3, 101 leaves, 1 step):

$$
-\frac{\operatorname{arctanh}\left(\frac{\left(2^{1 / 3}+\left(b x^{2}-2\right)^{1 / 3}\right)^{2} 2^{5 / 6}}{6 x \sqrt{b}}\right) \sqrt{b} 2^{1 / 6}}{24 d}+\frac{\operatorname{arctanh}\left(\frac{x \sqrt{b} \sqrt{2}}{6}\right) \sqrt{b} 2^{1 / 6}}{24 d}+\frac{\arctan \left(\frac{2^{1 / 6}\left(2^{1 / 3}+\left(b x^{2}-2\right)^{1 / 3}\right) \sqrt{3}}{x \sqrt{b}}\right) \sqrt{b} 2^{1 / 6} \sqrt{3}}{24 d}
$$

Result(type 8, 26 leaves):

$$
\int \frac{1}{\left(b x^{2}-2\right)^{1 / 3}\left(-\frac{18 d}{b}+d x^{2}\right)} \mathrm{d} x
$$

Problem 46: Unable to integrate problem.

$$
\int \frac{1}{\left(3 x^{2}+2\right)^{1 / 3}\left(d x^{2}+6 d\right)} d x
$$

Optimal(type 3, 90 leaves, 1 step):

$$
-\frac{\operatorname{arctanh}\left(\frac{2^{1 / 6}\left(2^{1 / 3}-\left(3 x^{2}+2\right)^{1 / 3}\right)}{x}\right) 2^{1 / 6}}{8 d}+\frac{\arctan \left(\frac{\left(2^{1 / 3}-\left(3 x^{2}+2\right)^{1 / 3}\right)^{2} 2^{5 / 6} \sqrt{3}}{18 x}\right) 2^{1 / 6} \sqrt{3}}{24 d}+\frac{\arctan \left(\frac{x \sqrt{6}}{6}\right) 2^{1 / 6} \sqrt{3}}{24 d}
$$

Result(type 8, 23 leaves):

$$
\int \frac{1}{\left(3 x^{2}+2\right)^{1 / 3}\left(d x^{2}+6 d\right)} \mathrm{d} x
$$

Problem 53: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{\left(b x^{2}+a\right)^{5 / 2} \sqrt{d x^{2}+c}} d x
$$

Optimal(type 4, 295 leaves, 4 steps):

$$
-\frac{\left.(-3 a d+b c) \sqrt{\frac{1}{1+\frac{d x^{2}}{c}}} \sqrt{1+\frac{d x^{2}}{c}} \text { EllipticF } \frac{x \sqrt{d}}{\sqrt{c} \sqrt{1+\frac{d x^{2}}{c}}}, \sqrt{1-\frac{b c}{a d}}\right) \sqrt{c} \sqrt{d} \sqrt{b x^{2}+a}}{3 a^{2}(-a d+b c)^{2} \sqrt{\frac{c\left(b x^{2}+a\right)}{a\left(d x^{2}+c\right)}} \sqrt{d x^{2}+c}}+\frac{b x \sqrt{d x^{2}+c}}{3 a(-a d+b c)\left(b x^{2}+a\right)^{3 / 2}}
$$

$$
+\frac{\left.2(-2 a d+b c) \sqrt{\frac{1}{1+\frac{b x^{2}}{a}}} \sqrt{1+\frac{b x^{2}}{a}} \text { EllipticE } \frac{x \sqrt{b}}{\sqrt{a} \sqrt{1+\frac{b x^{2}}{a}}}, \sqrt{1-\frac{a d}{b c}}\right) \sqrt{b} \sqrt{d x^{2}+c}}{3 a^{3 / 2}(-a d+b c)^{2} \sqrt{b x^{2}+a} \sqrt{\frac{a\left(d x^{2}+c\right)}{c\left(b x^{2}+a\right)}}}
$$

Result(type 4, 751 leaves):

$$
\begin{aligned}
& \frac{1}{3 \sqrt{d x^{2}+c}(a d-b c)^{2} \sqrt{-\frac{b}{a}} a^{2}\left(b x^{2}+a\right)^{3 / 2}}\left(- 4 x ^ { 5 } a b ^ { 2 } d ^ { 2 } \sqrt { - \frac { b } { a } } + 2 x ^ { 5 } b ^ { 3 } c d \sqrt { - \frac { b } { a } } + 3 \text { EllipticF } \left(x \sqrt{-\frac{b}{a}},\right.\right. \\
& \left.\sqrt{\frac{a d}{b c}}\right) x^{2} a^{2} b d^{2} \sqrt{\frac{b x^{2}+a}{a}} \sqrt{\frac{d x^{2}+c}{c}}-5 \operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) x^{2} a b^{2} c d \sqrt{\frac{b x^{2}+a}{a}} \sqrt{\frac{d x^{2}+c}{c}}+2 \text { EllipticF }\left(x \sqrt{-\frac{b}{a}},\right. \\
& \left.\sqrt{\frac{a d}{b c}}\right) x^{2} b^{3} c^{2} \sqrt{\frac{b x^{2}+a}{a}} \sqrt{\frac{d x^{2}+c}{c}}+4 \text { EllipticE }\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) x^{2} a b^{2} c d \sqrt{\frac{b x^{2}+a}{a}} \sqrt{\frac{d x^{2}+c}{c}}-2 \text { EllipticE }\left(x \sqrt{-\frac{b}{a}},\right. \\
& \left.\sqrt{\frac{a d}{b c}}\right) x^{2} b^{3} c^{2} \sqrt{\frac{b x^{2}+a}{a}} \sqrt{\frac{d x^{2}+c}{c}}-5 x^{3} a^{2} b d^{2} \sqrt{-\frac{b}{a}}-x^{3} a b^{2} c d \sqrt{-\frac{b}{a}}+2 x^{3} b^{3} c^{2} \sqrt{-\frac{b}{a}}+3 \text { EllipticF }\left(x \sqrt{-\frac{b}{a}}\right. \text {, } \\
& \left.\sqrt{\frac{a d}{b c}}\right) a^{3} d^{2} \sqrt{\frac{b x^{2}+a}{a}} \sqrt{\frac{d x^{2}+c}{c}}-5 \text { EllipticF }\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) a^{2} b c d \sqrt{\frac{b x^{2}+a}{a}} \sqrt{\frac{d x^{2}+c}{c}}+2 \text { EllipticF }\left(x \sqrt{-\frac{b}{a}}\right. \text {, } \\
& \left.\sqrt{\frac{a d}{b c}}\right) a b^{2} c^{2} \sqrt{\frac{b x^{2}+a}{a}} \sqrt{\frac{d x^{2}+c}{c}}+4 \text { EllipticE }\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) a^{2} b c d \sqrt{\frac{b x^{2}+a}{a}} \sqrt{\frac{d x^{2}+c}{c}}-2 \text { EllipticE }\left(x \sqrt{-\frac{b}{a}}\right. \text {, } \\
& \left.\left.\sqrt{\frac{a d}{b c}}\right) a b^{2} c^{2} \sqrt{\frac{b x^{2}+a}{a}} \sqrt{\frac{d x^{2}+c}{c}}-5 x a^{2} b c d \sqrt{-\frac{b}{a}}+3 x a b^{2} c^{2} \sqrt{-\frac{b}{a}}\right)
\end{aligned}
$$

Problem 71: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{-b x^{2}-a}}{\sqrt{-d x^{2}+c}} \mathrm{~d} x
$$

Optimal(type 4, 75 leaves, 3 steps):

$$
\frac{\text { EllipticE }\left(\frac{x \sqrt{d}}{\sqrt{c}}, \sqrt{-\frac{b c}{a d}}\right) \sqrt{c} \sqrt{-b x^{2}-a} \sqrt{1-\frac{d x^{2}}{c}}}{\sqrt{d} \sqrt{1+\frac{b x^{2}}{a}} \sqrt{-d x^{2}+c}}
$$

Result(type 4, 170 leaves):

$$
\begin{aligned}
& \frac{1}{\left(b d x^{4}+a d x^{2}-b c x^{2}-a c\right) \sqrt{-\frac{b}{a}} d}\left(\left(- a \text { EllipticF } ( x \sqrt { - \frac { b } { a } } , \sqrt { - \frac { a d } { b c } } ) d - b c \text { EllipticF } ( x \sqrt { - \frac { b } { a } } , \sqrt { - \frac { a d } { b c } } ) + b c \text { EllipticE } \left(x \sqrt{-\frac{b}{a}}\right.\right.\right. \\
& \left.\left.\left.\sqrt{-\frac{a d}{b c}}\right)\right) \sqrt{-b x^{2}-a} \sqrt{-d x^{2}+c} \sqrt{\frac{b x^{2}+a}{a}} \sqrt{-\frac{d x^{2}-c}{c}}\right)
\end{aligned}
$$

Problem 72: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{-b x^{2}+a}}{\sqrt{d x^{2}-c}} d x
$$

Optimal (type 4, 74 leaves, 3 steps):

$$
\frac{\text { EllipticE }\left(\frac{x \sqrt{d}}{\sqrt{c}}, \sqrt{\frac{b c}{a d}}\right) \sqrt{c} \sqrt{-b x^{2}+a} \sqrt{1-\frac{d x^{2}}{c}}}{\sqrt{d} \sqrt{1-\frac{b x^{2}}{a}} \sqrt{d x^{2}-c}}
$$

Result (type 4, 165 leaves):

$$
\frac{\left(-a \text { EllipticF }\left(\sqrt{\frac{b}{a}} x, \sqrt{\frac{a d}{b c}}\right) d+b c \text { EllipticF }\left(\sqrt{\frac{b}{a}} x, \sqrt{\frac{a d}{b c}}\right)-b c \text { EllipticE }\left(\sqrt{\frac{b}{a}} x, \sqrt{\frac{a d}{b c}}\right)\right) \sqrt{-b x^{2}+a} \sqrt{d x^{2}-c} \sqrt{-\frac{d x^{2}-c}{c}} \sqrt{-\frac{b x^{2}-a}{a}}}{\left(b d x^{4}-a d x^{2}-b c x^{2}+a c\right) \sqrt{\frac{b}{a}} d}
$$

Problem 74: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{-d x^{2}+c}}{\sqrt{-b x^{2}+a}} d x
$$

Optimal (type 4, 73 leaves, 3 steps):

$$
\frac{\operatorname{EllipticE}\left(\frac{x \sqrt{b}}{\sqrt{a}}, \sqrt{\frac{a d}{b c}}\right) \sqrt{a} \sqrt{1-\frac{b x^{2}}{a}} \sqrt{-d x^{2}+c}}{\sqrt{b} \sqrt{-b x^{2}+a} \sqrt{1-\frac{d x^{2}}{c}}}
$$

Result(type 4, 163 leaves):

$$
\frac{\left(-a d \text { EllipticF }\left(x \sqrt{\frac{d}{c}}, \sqrt{\frac{b c}{a d}}\right)+c \text { EllipticF }\left(x \sqrt{\frac{d}{c}}, \sqrt{\frac{b c}{a d}}\right) b+a d \operatorname{EllipticE}\left(x \sqrt{\frac{d}{c}}, \sqrt{\frac{b c}{a d}}\right)\right) \sqrt{-b x^{2}+a} \sqrt{-d x^{2}+c} \sqrt{-\frac{d x^{2}-c}{c}} \sqrt{-\frac{b x^{2}-a}{a}}}{b \sqrt{\frac{d}{c}}\left(b d x^{4}-a d x^{2}-b c x^{2}+a c\right)}
$$

Problem 75: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{d x^{2}-c}}{\sqrt{b x^{2}-a}} d x
$$

Optimal(type 4, 75 leaves, 3 steps):

$$
\frac{\text { EllipticE }\left(\frac{x \sqrt{b}}{\sqrt{a}}, \sqrt{\frac{a d}{b c}}\right) \sqrt{a} \sqrt{1-\frac{b x^{2}}{a}} \sqrt{d x^{2}-c}}{\sqrt{b} \sqrt{b x^{2}-a} \sqrt{1-\frac{d x^{2}}{c}}}
$$

Result(type 4, 166 leaves):

$$
\frac{\sqrt{d x^{2}-c} \sqrt{b x^{2}-a} \sqrt{-\frac{d x^{2}-c}{c}} \sqrt{-\frac{b x^{2}-a}{a}}\left(a d \text { EllipticF }\left(x \sqrt{\frac{d}{c}}, \sqrt{\frac{b c}{a d}}\right)-c \text { EllipticF }\left(x \sqrt{\frac{d}{c}}, \sqrt{\frac{b c}{a d}}\right) b-a d \text { EllipticE }\left(x \sqrt{\frac{d}{c}}, \sqrt{\frac{b c}{a d}}\right)\right)}{b \sqrt{\frac{d}{c}}\left(b d x^{4}-a d x^{2}-b c x^{2}+a c\right)}
$$

Problem 80: Unable to integrate problem.

$$
\int \frac{\sqrt{1+\frac{2 c x^{2}}{b-\sqrt{-4 a c+b^{2}}}}}{\sqrt{1+\frac{2 c x^{2}}{b+\sqrt{-4 a c+b^{2}}}}} d x
$$

Optimal(type 4, 457 leaves, 4 steps):
$\frac{x \sqrt{1+\frac{2 c x^{2}}{b-\sqrt{-4 a c+b^{2}}}}}{\sqrt{1+\frac{2 c x^{2}}{b+\sqrt{-4 a c+b^{2}}}}}$

$$
\begin{aligned}
& -\frac{1}{2 \sqrt{c} \sqrt{\frac{1+\frac{2-\sqrt{-4 a c+b^{2}}}{1+\frac{2 c x^{2}}{b+\sqrt{-4 a c+b^{2}}}}}{}\left(\sqrt{\frac{1}{1+\frac{2 c x^{2}}{b+\sqrt{-4 a c+b^{2}}}}}\right.} \text { EllipticE }\left(\frac{x \sqrt{2} \sqrt{c}}{\sqrt{b+\sqrt{-4 a c+b^{2}}} \sqrt{1+\frac{2 c x^{2}}{b+\sqrt{-4 a c+b^{2}}}}}, ~, ~, ~\right.} \\
& \left.\left.\sqrt{-\frac{2 \sqrt{-4 a c+b^{2}}}{b-\sqrt{-4 a c+b^{2}}}}\right) \sqrt{1+\frac{2 c x^{2}}{b-\sqrt{-4 a c+b^{2}}}} \sqrt{b+\sqrt{-4 a c+b^{2}}} \sqrt{2}\right) \\
& +\frac{1}{2 \sqrt{c} \sqrt{\frac{1+\frac{2-\sqrt{-4 a c+b^{2}}}{\frac{2 c x^{2}}{1+\frac{2 c x^{2}}{b+\sqrt{-4 a c+b^{2}}}}}}{}\left(\sqrt{\frac{1}{1+\frac{2 c x^{2}}{b+\sqrt{-4 a c+b^{2}}}}}\right.} \text { EllipticF } \frac{x \sqrt{2} \sqrt{c}}{\sqrt{b+\sqrt{-4 a c+b^{2}}} \sqrt{1+\frac{2 c x^{2}}{b+\sqrt{-4 a c+b^{2}}}}},} \\
& \left.\left.\sqrt{-\frac{2 \sqrt{-4 a c+b^{2}}}{b-\sqrt{-4 a c+b^{2}}}}\right) \sqrt{1+\frac{2 c x^{2}}{b-\sqrt{-4 a c+b^{2}}}} \sqrt{b+\sqrt{-4 a c+b^{2}}} \sqrt{2}\right)
\end{aligned}
$$

Result(type 8, 53 leaves):

$$
\int \frac{\sqrt{1+\frac{2 c x^{2}}{b-\sqrt{-4 a c+b^{2}}}}}{\sqrt{1+\frac{2 c x^{2}}{b+\sqrt{-4 a c+b^{2}}}}} d x
$$

Problem 81: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{\sqrt{x^{2}-1} \sqrt{7+x^{2}-4 \sqrt{3}}} d x
$$

Optimal(type 4, 37 leaves, 2 steps):

$$
\frac{\text { EllipticF }(x, \mathrm{I} \sqrt{3}+2 \mathrm{I}) \sqrt{-x^{2}+1}}{\sqrt{x^{2}-1}(2-\sqrt{3})}
$$

Result(type 4, 116 leaves):

$$
\frac{-\mathrm{IEllipticF}\left(\frac{\mathrm{I} x}{-2+\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{-x^{2}+1} \sqrt{-\left(-x^{2}+4 \sqrt{3}-7\right)(-4 \sqrt{3}+7)}(-2+\sqrt{3}) \sqrt{x^{2}-1} \sqrt{7+x^{2}-4 \sqrt{3}}}{(4 \sqrt{3}-7)\left(-x^{4}+4 \sqrt{3} x^{2}-6 x^{2}-4 \sqrt{3}+7\right)}
$$

Problem 82: Unable to integrate problem.

$$
\int \frac{1}{\left(-3 x^{2}+a\right)^{1 / 4}\left(-3 x^{2}+2 a\right)} \mathrm{d} x
$$

Optimal(type 3, 90 leaves, 1 step):

$$
\frac{\arctan \left(\frac{a^{3 / 4}\left(1-\frac{\sqrt{-3 x^{2}+a}}{\sqrt{a}}\right) \sqrt{3}}{3 x\left(-3 x^{2}+a\right)^{1 / 4}}\right) \sqrt{3}}{6 a^{3 / 4}}+\frac{\operatorname{arctanh}\left(\frac{a^{3 / 4}\left(1+\frac{\sqrt{-3 x^{2}+a}}{\sqrt{a}}\right) \sqrt{3}}{3 x\left(-3 x^{2}+a\right)^{1 / 4}}\right) \sqrt{3}}{6 a^{3 / 4}}
$$

Result(type 8, 23 leaves):

$$
\int \frac{1}{\left(-3 x^{2}+a\right)^{1 / 4}\left(-3 x^{2}+2 a\right)} \mathrm{d} x
$$

Problem 83: Unable to integrate problem.

$$
\int \frac{1}{\left(3 x^{2}-2\right)\left(3 x^{2}-1\right)^{1 / 4}} \mathrm{~d} x
$$

Optimal(type 3, 43 leaves, 1 step):

$$
-\frac{\arctan \left(\frac{x \sqrt{6}}{2\left(3 x^{2}-1\right)^{1 / 4}}\right) \sqrt{6}}{12}-\frac{\operatorname{arctanh}\left(\frac{x \sqrt{6}}{2\left(3 x^{2}-1\right)^{1 / 4}}\right) \sqrt{6}}{12}
$$

Result(type 8, 21 leaves):

$$
\int \frac{1}{\left(3 x^{2}-2\right)\left(3 x^{2}-1\right)^{1 / 4}} \mathrm{~d} x
$$

Problem 84: Unable to integrate problem.

$$
\int \frac{1}{\left(3 x^{2}-2 a\right)\left(3 x^{2}-a\right)^{1 / 4}} \mathrm{~d} x
$$

Optimal(type 3, 59 leaves, 1 step):

$$
-\frac{\arctan \left(\frac{x \sqrt{6}}{2 a^{1 / 4}\left(3 x^{2}-a\right)^{1 / 4}}\right) \sqrt{6}}{12 a^{3 / 4}}-\frac{\operatorname{arctanh}\left(\frac{x \sqrt{6}}{2 a^{1 / 4}\left(3 x^{2}-a\right)^{1 / 4}}\right) \sqrt{6}}{12 a^{3 / 4}}
$$

Result(type 8, 25 leaves):

$$
\int \frac{1}{\left(3 x^{2}-2 a\right)\left(3 x^{2}-a\right)^{1 / 4}} \mathrm{~d} x
$$

Problem 85: Unable to integrate problem.

$$
\int \frac{1}{\left(b x^{2}+a\right)^{5 / 4}\left(d x^{2}+c\right)} \mathrm{d} x
$$

Optimal(type 4, 216 leaves, 7 steps):
$\frac{2\left(1+\frac{b x^{2}}{a}\right)^{1 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{b}}{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)}(-a d+b c)\left(b x^{2}+a\right)^{1 / 4} \sqrt{a}$

$$
+\frac{a^{1 / 4} \text { EllipticPi }\left(\frac{\left(b x^{2}+a\right)^{1 / 4}}{a^{1 / 4}},-\frac{\sqrt{a} \sqrt{d}}{\sqrt{a d-b c}}, \mathrm{I}\right) \sqrt{d} \sqrt{-\frac{b x^{2}}{a}}}{(a d-b c)^{3 / 2} x}-\frac{a^{1 / 4} \operatorname{EllipticPi}\left(\frac{\left(b x^{2}+a\right)^{1 / 4}}{a^{1 / 4}}, \frac{\sqrt{a} \sqrt{d}}{\sqrt{a d-b c}}, \mathrm{I}\right) \sqrt{d} \sqrt{-\frac{b x^{2}}{a}}}{(a d-b c)^{3 / 2} x}
$$

Result(type 8, 21 leaves):

$$
\int \frac{1}{\left(b x^{2}+a\right)^{5 / 4}\left(d x^{2}+c\right)} \mathrm{d} x
$$

Problem 86: Unable to integrate problem.

$$
\int \frac{1}{\left(b x^{2}+a\right)^{11 / 4}\left(d x^{2}+c\right)} \mathrm{d} x
$$

Optimal(type 4, 285 leaves, 10 steps):
$\frac{2 b x}{7 a(-a d+b c)\left(b x^{2}+a\right)^{7 / 4}}+\frac{2 b(-12 a d+5 b c) x}{21 a^{2}(-a d+b c)^{2}\left(b x^{2}+a\right)^{3 / 4}}$

$$
\begin{aligned}
& +\frac{2(-12 a d+5 b c)\left(1+\frac{b x^{2}}{a}\right)^{3 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{b}}{21 \cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) a^{3 / 2}(-a d+b c)^{2}\left(b x^{2}+a\right)^{3 / 4}} \\
& +\frac{a^{1 / 4} d^{2} \operatorname{EllipticPi}\left(\frac{\left(b x^{2}+a\right)^{1 / 4}}{a^{1 / 4}},-\frac{\sqrt{a} \sqrt{d}}{\sqrt{a d-b c}}, \mathrm{I}\right) \sqrt{-\frac{b x^{2}}{a}}}{(-a d+b c)^{3} x}+\frac{a^{1 / 4} d^{2} \operatorname{EllipticPi}\left(\frac{\left(b x^{2}+a\right)^{1 / 4}}{a^{1 / 4}}, \frac{\sqrt{a} \sqrt{d}}{\sqrt{a d-b c}}, \mathrm{I}\right) \sqrt{-\frac{b x^{2}}{a}}}{(-a d+b c)^{3} x}
\end{aligned}
$$

Result(type 8, 21 leaves):

$$
\int \frac{1}{\left(b x^{2}+a\right)^{11 / 4}\left(d x^{2}+c\right)} \mathrm{d} x
$$

Problem 87: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{5 / 4}}{\left(d x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 260 leaves, 9 steps):
$-\frac{(-a d+b c) x\left(b x^{2}+a\right)^{1 / 4}}{2 c d\left(d x^{2}+c\right)}+\frac{(a d+3 b c)\left(1+\frac{b x^{2}}{a}\right)^{3 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{a} \sqrt{b}}{2 \cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) c d^{2}\left(b x^{2}+a\right)^{3 / 4}}$
$-\frac{a^{1 / 4}(2 a d+3 b c) \text { EllipticPi }\left(\frac{\left(b x^{2}+a\right)^{1 / 4}}{a^{1 / 4}},-\frac{\sqrt{a} \sqrt{d}}{\sqrt{a d-b c}}, \mathrm{I}\right) \sqrt{-\frac{b x^{2}}{a}}}{4 c d^{2} x}$
$-\frac{a^{1 / 4}(2 a d+3 b c) \text { EllipticPi }\left(\frac{\left(b x^{2}+a\right)^{1 / 4}}{a^{1 / 4}}, \frac{\sqrt{a} \sqrt{d}}{\sqrt{a d-b c}}, \mathrm{I}\right) \sqrt{-\frac{b x^{2}}{a}}}{4 c d^{2} x}$
Result(type 8, 21 leaves):

$$
\int \frac{\left(b x^{2}+a\right)^{5 / 4}}{\left(d x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Problem 88: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{3 / 4}}{\left(d x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 278 leaves, 9 steps):

$$
\begin{aligned}
& -\frac{b x}{2 c d\left(b x^{2}+a\right)^{1 / 4}}+\frac{x\left(b x^{2}+a\right)^{3 / 4}}{2 c\left(d x^{2}+c\right)}+\frac{\left(1+\frac{b x^{2}}{a}\right)^{1 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{a} \sqrt{b}}{2 \cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) c d\left(b x^{2}+a\right)^{1 / 4}} \\
& \quad+\frac{a^{1 / 4}(2 a d+b c) \operatorname{EllipticPi}\left(\frac{\left(b x^{2}+a\right)^{1 / 4}}{a^{1 / 4}},-\frac{\sqrt{a} \sqrt{d}}{\sqrt{a d-b c}}, \mathrm{I}\right) \sqrt{-\frac{b x^{2}}{a}}}{4 c d^{3 / 2} x \sqrt{a d-b c}}-\frac{a^{1 / 4}(2 a d+b c) \operatorname{EllipticPi}\left(\frac{\left(b x^{2}+a\right)^{1 / 4}}{a^{1 / 4}}, \frac{\sqrt{a} \sqrt{d}}{\sqrt{a d-b c}}, \mathrm{I}\right) \sqrt{-\frac{b x^{2}}{a}}}{4 c d^{3 / 2} x \sqrt{a d-b c}}
\end{aligned}
$$

Result(type 8, 21 leaves):

$$
\int \frac{\left(b x^{2}+a\right)^{3 / 4}}{\left(d x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Problem 89: Unable to integrate problem.

$$
\int \frac{1}{\left(b x^{2}+a\right)^{1 / 4}\left(d x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 305 leaves, 9 steps):

$$
\frac{b x}{2 c(-a d+b c)\left(b x^{2}+a\right)^{1 / 4}}-\frac{d x\left(b x^{2}+a\right)^{3 / 4}}{2 c(-a d+b c)\left(d x^{2}+c\right)}
$$

$$
-\frac{\left(1+\frac{b x^{2}}{a}\right)^{1 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \text { EllipticE }\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{a} \sqrt{b}}{2 \cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) c(-a d+b c)\left(b x^{2}+a\right)^{1 / 4}}
$$

$$
-\frac{a^{1 / 4}(-2 a d+3 b c) \text { EllipticPi }\left(\frac{\left(b x^{2}+a\right)^{1 / 4}}{a^{1 / 4}},-\frac{\sqrt{a} \sqrt{d}}{\sqrt{a d-b c}}, \mathrm{I}\right) \sqrt{-\frac{b x^{2}}{a}}}{}
$$

$$
4 c(a d-b c)^{3 / 2} x \sqrt{d}
$$

$$
+\frac{a^{1 / 4}(-2 a d+3 b c) \text { EllipticPi }\left(\frac{\left(b x^{2}+a\right)^{1 / 4}}{a^{1 / 4}}, \frac{\sqrt{a} \sqrt{d}}{\sqrt{a d-b c}}, \mathrm{I}\right) \sqrt{-\frac{b x^{2}}{a}}}{4 c(a d-b c)^{3 / 2} x \sqrt{d}}
$$

Result(type 8, 21 leaves):

$$
\int \frac{1}{\left(b x^{2}+a\right)^{1 / 4}\left(d x^{2}+c\right)^{2}} d x
$$

Problem 90: Unable to integrate problem.

$$
\int \frac{1}{\left(b x^{2}+a\right)^{5 / 4}\left(d x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 287 leaves, 10 steps):
$-\frac{d x}{2 c(-a d+b c)\left(b x^{2}+a\right)^{1 / 4}\left(d x^{2}+c\right)}+\frac{(a d+4 b c)\left(1+\frac{b x^{2}}{a}\right)^{1 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)^{2}}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{b}}{2 \cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) c(-a d+b c)^{2}\left(b x^{2}+a\right)^{1 / 4} \sqrt{a}}$
$-\xrightarrow{a^{1 / 4}(-2 a d+7 b c) \text { EllipticPi }\left(\frac{\left(b x^{2}+a\right)^{1 / 4}}{a^{1 / 4}},-\frac{\sqrt{a} \sqrt{d}}{\sqrt{a d-b c}}, \mathrm{I}\right) \sqrt{d} \sqrt{-\frac{b x^{2}}{a}}}$
$4 c(a d-b c)^{5 / 2} x$
$+\frac{a^{1 / 4}(-2 a d+7 b c) \text { EllipticPi }\left(\frac{\left(b x^{2}+a\right)^{1 / 4}}{a^{1 / 4}}, \frac{\sqrt{a} \sqrt{d}}{\sqrt{a d-b c}}, \mathrm{I}\right) \sqrt{d} \sqrt{-\frac{b x^{2}}{a}}}{4 c(a d-b c)^{5 / 2} x}$
Result(type 8, 21 leaves):

$$
\int \frac{1}{\left(b x^{2}+a\right)^{5 / 4}\left(d x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Problem 91: Unable to integrate problem.

$$
\int \frac{1}{\left(b x^{2}+a\right)^{9 / 4}\left(d x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 340 leaves, 11 steps):
$\frac{b(5 a d+4 b c) x}{10 a c(-a d+b c)^{2}\left(b x^{2}+a\right)^{5 / 4}}-\frac{d x}{2 c(-a d+b c)\left(b x^{2}+a\right)^{5 / 4}\left(d x^{2}+c\right)}$

$$
\begin{aligned}
& +\frac{\left(-5 a^{2} d^{2}-52 a c b d+12 b^{2} c^{2}\right)\left(1+\frac{b x^{2}}{a}\right)^{1 / 4} \sqrt{\cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)^{2}}{2}\right)^{2}} \text { EllipticE }\left(\sin \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{b}}{10 \cos \left(\frac{\arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) a^{3 / 2} c(-a d+b c)^{3}\left(b x^{2}+a\right)^{1 / 4}} \\
& -\frac{a^{1 / 4} d^{3 / 2}(-2 a d+11 b c) \text { EllipticPi }\left(\frac{\left(b x^{2}+a\right)^{1 / 4}}{a^{1 / 4}},-\frac{\sqrt{a} \sqrt{d}}{\sqrt{a d-b c}}, \mathrm{I}\right) \sqrt{-\frac{b x^{2}}{a}}}{4 c(a d-b c)^{7 / 2} x} \\
& +\frac{a^{1 / 4} d^{3 / 2}(-2 a d+11 b c) \text { EllipticPi }\left(\frac{\left(b x^{2}+a\right)^{1 / 4}}{a^{1 / 4}}, \frac{\sqrt{a} \sqrt{d}}{\sqrt{a d-b c}}, \mathrm{I}\right) \sqrt{-\frac{b x^{2}}{a}}}{4 c(a d-b c)^{7 / 2} x}
\end{aligned}
$$

Result(type 8, 21 leaves):

$$
\int \frac{1}{\left(b x^{2}+a\right)^{9 / 4}\left(d x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Problem 92: Unable to integrate problem.

$$
\int\left(b x^{2}+a\right)^{p}\left(d x^{2}+c\right)^{2} \mathrm{~d} x
$$

Optimal(type 5, 174 leaves, 4 steps):

$$
\begin{aligned}
& -\frac{d(3 a d-b c(7+2 p)) x\left(b x^{2}+a\right)^{1+p}}{b^{2}(2 p+3)(2 p+5)}+\frac{d x\left(b x^{2}+a\right)^{1+p}\left(d x^{2}+c\right)}{b(2 p+5)} \\
& \quad+\frac{\left(3 a^{2} d^{2}-2 a b c d(2 p+5)+b^{2} c^{2}\left(4 p^{2}+16 p+15\right)\right) x\left(b x^{2}+a\right)^{p} \operatorname{hypergeom}\left(\left[\frac{1}{2},-p\right],\left[\frac{3}{2}\right],-\frac{b x^{2}}{a}\right)}{b^{2}(2 p+3)(2 p+5)\left(1+\frac{b x^{2}}{a}\right)^{p}}
\end{aligned}
$$

Result(type 8, 21 leaves):

$$
\int\left(b x^{2}+a\right)^{p}\left(d x^{2}+c\right)^{2} \mathrm{~d} x
$$

Problem 93: Unable to integrate problem.

$$
\int\left(b x^{2}+a\right)^{p} \mathrm{~d} x
$$

Optimal(type 5, 42 leaves, 2 steps):

$$
\frac{x\left(b x^{2}+a\right)^{p} \text { hypergeom }\left(\left[\frac{1}{2},-p\right],\left[\frac{3}{2}\right],-\frac{b x^{2}}{a}\right)}{\left(1+\frac{b x^{2}}{a}\right)^{p}}
$$

Result(type 8, 11 leaves):

$$
\int\left(b x^{2}+a\right)^{p} \mathrm{~d} x
$$

Problem 94: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{p}}{\left(d x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Optimal(type 6, 53 leaves, 2 steps):

$$
\frac{x\left(b x^{2}+a\right)^{p} \text { AppellF1 }\left(\frac{1}{2},-p, 2, \frac{3}{2},-\frac{b x^{2}}{a},-\frac{d x^{2}}{c}\right)}{c^{2}\left(1+\frac{b x^{2}}{a}\right)^{p}}
$$

Result(type 8, 21 leaves):

$$
\int \frac{\left(b x^{2}+a\right)^{p}}{\left(d x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Problem 95: Unable to integrate problem.

$$
\int \frac{\left(b x^{2}+a\right)^{p}}{\left(d x^{2}+c\right)^{3}} \mathrm{~d} x
$$

Optimal(type 6, 53 leaves, 2 steps):

$$
\frac{x\left(b x^{2}+a\right)^{p} \text { AppellF1 }\left(\frac{1}{2},-p, 3, \frac{3}{2},-\frac{b x^{2}}{a},-\frac{d x^{2}}{c}\right)}{c^{3}\left(1+\frac{b x^{2}}{a}\right)^{p}}
$$

Result(type 8, 21 leaves):

$$
\int \frac{\left(b x^{2}+a\right)^{p}}{\left(d x^{2}+c\right)^{3}} \mathrm{~d} x
$$

Test results for the 301 problems in "1.1.2.4 (e x) ^m (a+b $\left.x^{\wedge} 2\right)^{\wedge} p\left(c+d x^{\wedge} 2\right)^{\wedge} q . t x t "$
Problem 11: Result more than twice size of optimal antiderivative.

$$
\int x^{3}\left(b x^{2}+a\right)^{5}\left(B x^{2}+A\right) \mathrm{d} x
$$

Optimal(type 1, 61 leaves, 3 steps):

$$
-\frac{a(A b-a B)\left(b x^{2}+a\right)^{6}}{12 b^{3}}+\frac{(A b-2 a B)\left(b x^{2}+a\right)^{7}}{14 b^{3}}+\frac{B\left(b x^{2}+a\right)^{8}}{16 b^{3}}
$$

Result(type 1, 123 leaves):

$$
\begin{aligned}
& \frac{b^{5} B x^{16}}{16}+\frac{\left(b^{5} A+5 a b^{4} B\right) x^{14}}{14}+\frac{\left(5 a b^{4} A+10 a^{2} b^{3} B\right) x^{12}}{12}+\frac{\left(10 a^{2} b^{3} A+10 a^{3} b^{2} B\right) x^{10}}{10}+\frac{\left(10 a^{3} b^{2} A+5 a^{4} b B\right) x^{8}}{8}+\frac{\left(5 a^{4} b A+a^{5} B\right) x^{6}}{6} \\
& \quad+\frac{a^{5} A x^{4}}{4}
\end{aligned}
$$

Problem 16: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(b x^{2}+a\right)^{5}\left(B x^{2}+A\right)}{x^{15}} d x
$$

Optimal(type 1, 44 leaves, 3 steps):

$$
-\frac{A\left(b x^{2}+a\right)^{6}}{14 a x^{14}}+\frac{(A b-7 a B)\left(b x^{2}+a\right)^{6}}{84 a^{2} x^{12}}
$$

Result(type 1, 103 leaves):

$$
-\frac{b^{5} B}{2 x^{2}}-\frac{a^{5} A}{14 x^{14}}-\frac{b^{4}(A b+5 a B)}{4 x^{4}}-\frac{5 a^{2} b^{2}(A b+a B)}{4 x^{8}}-\frac{a^{3} b(2 A b+a B)}{2 x^{10}}-\frac{a^{4}(5 A b+a B)}{12 x^{12}}-\frac{5 a b^{3}(A b+2 a B)}{6 x^{6}}
$$

Problem 33: Result more than twice size of optimal antiderivative.

$$
\int \frac{2 x^{2}+1}{x^{5}\left(x^{2}+1\right)^{3}} d x
$$

Optimal(type 1, 12 leaves, 2 steps):

$$
-\frac{1}{4 x^{4}\left(x^{2}+1\right)^{2}}
$$

Result(type 1, 29 leaves):

$$
-\frac{1}{4\left(x^{2}+1\right)^{2}}-\frac{1}{2\left(x^{2}+1\right)}-\frac{1}{4 x^{4}}+\frac{1}{2 x^{2}}
$$

Problem 58: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{2}\left(d x^{2}+c\right)^{3}}{b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 3, 105 leaves, 3 steps):

$$
\frac{(-a d+b c)^{3} x}{b^{4}}+\frac{d\left(a^{2} d^{2}-3 a c b d+3 b^{2} c^{2}\right) x^{3}}{3 b^{3}}+\frac{d^{2}(-a d+3 b c) x^{5}}{5 b^{2}}+\frac{d^{3} x^{7}}{7 b}-\frac{(-a d+b c)^{3} \arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right) \sqrt{a}}{b^{9 / 2}}
$$

Result(type 3, 217 leaves):

$$
\begin{aligned}
& \frac{d^{3} x^{7}}{7 b}-\frac{x^{5} a d^{3}}{5 b^{2}}+\frac{3 x^{5} c d^{2}}{5 b}+\frac{x^{3} a^{2} d^{3}}{3 b^{3}}-\frac{x^{3} a c d^{2}}{b^{2}}+\frac{x^{3} c^{2} d}{b}-\frac{a^{3} d^{3} x}{b^{4}}+\frac{3 a^{2} c d^{2} x}{b^{3}}-\frac{3 a c^{2} d x}{b^{2}}+\frac{c^{3} x}{b}+\frac{a^{4} \arctan \left(\frac{b x}{\sqrt{a b}}\right) d^{3}}{b^{4} \sqrt{a b}} \\
&- \frac{3 a^{3} \arctan \left(\frac{b x}{\sqrt{a b}}\right) c d^{2}}{b^{3} \sqrt{a b}}+\frac{3 a^{2} \arctan \left(\frac{b x}{\sqrt{a b}}\right) c^{2} d}{b^{2} \sqrt{a b}}-\frac{a \arctan \left(\frac{b x}{\sqrt{a b}}\right) c^{3}}{b \sqrt{a b}}
\end{aligned}
$$

Problem 75: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{3}\left(d x^{2}+c\right)^{3}}{\left(b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 107 leaves, 3 steps):

$$
\frac{3 d(-a d+b c)^{2} x^{2}}{2 b^{4}}+\frac{d^{2}(-2 a d+3 b c) x^{4}}{4 b^{3}}+\frac{d^{3} x^{6}}{6 b^{2}}+\frac{a(-a d+b c)^{3}}{2 b^{5}\left(b x^{2}+a\right)}+\frac{(-4 a d+b c)(-a d+b c)^{2} \ln \left(b x^{2}+a\right)}{2 b^{5}}
$$

Result(type 3, 228 leaves):

$$
\begin{aligned}
& \frac{d^{3} x^{6}}{6 b^{2}}-\frac{d^{3} x^{4} a}{2 b^{3}}+\frac{3 d^{2} x^{4} c}{4 b^{2}}+\frac{3 d^{3} x^{2} a^{2}}{2 b^{4}}-\frac{3 d^{2} x^{2} a c}{b^{3}}+\frac{3 d x^{2} c^{2}}{2 b^{2}}-\frac{2 \ln \left(b x^{2}+a\right) a^{3} d^{3}}{b^{5}}+\frac{9 \ln \left(b x^{2}+a\right) a^{2} d^{2} c}{2 b^{4}}-\frac{3 \ln \left(b x^{2}+a\right) a d c^{2}}{b^{3}}+\frac{\ln \left(b x^{2}+a\right) c^{3}}{2 b^{2}} \\
& \quad-\frac{a^{4} d^{3}}{2 b^{5}\left(b x^{2}+a\right)}+\frac{3 a^{3} d^{2} c}{2 b^{4}\left(b x^{2}+a\right)}-\frac{3 a^{2} d c^{2}}{2 b^{3}\left(b x^{2}+a\right)}+\frac{a c^{3}}{2 b^{2}\left(b x^{2}+a\right)}
\end{aligned}
$$

Problem 77: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(d x^{2}+c\right)^{3}}{\left(b x^{2}+a\right)^{2}} d x
$$

Optimal(type 3, 92 leaves, 4 steps):

$$
\frac{d^{2}(-2 a d+3 b c) x}{b^{3}}+\frac{d^{3} x^{3}}{3 b^{2}}+\frac{(-a d+b c)^{3} x}{2 a b^{3}\left(b x^{2}+a\right)}+\frac{(-a d+b c)^{2}(5 a d+b c) \arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2 a^{3 / 2} b^{7 / 2}}
$$

Result(type 3, 204 leaves):

$$
\begin{aligned}
& \frac{d^{3} x^{3}}{3 b^{2}}-\frac{2 d^{3} a x}{b^{3}}+\frac{3 d^{2} c x}{b^{2}}-\frac{x a^{2} d^{3}}{2 b^{3}\left(b x^{2}+a\right)}+\frac{3 x a c d^{2}}{2 b^{2}\left(b x^{2}+a\right)}-\frac{3 x c^{2} d}{2 b\left(b x^{2}+a\right)}+\frac{x c^{3}}{2 a\left(b x^{2}+a\right)}+\frac{5 a^{2} \arctan \left(\frac{b x}{\sqrt{a b}}\right) d^{3}}{2 b^{3} \sqrt{a b}}-\frac{9 a \arctan \left(\frac{b x}{\sqrt{a b}}\right) c d^{2}}{2 b^{2} \sqrt{a b}} \\
& \quad+\frac{3 \arctan \left(\frac{b x}{\sqrt{a b}}\right) c^{2} d}{2 b \sqrt{a b}}+\frac{\arctan \left(\frac{b x}{\sqrt{a b}}\right) c^{3}}{2 a \sqrt{a b}}
\end{aligned}
$$

Problem 83: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{2}}{\left(b x^{2}+a\right)^{2}\left(d x^{2}+c\right)^{3}} \mathrm{~d} x
$$

Optimal(type 3, 174 leaves, 6 steps):

$$
\begin{aligned}
& -\frac{3 d x}{4(-a d+b c)^{2}\left(d x^{2}+c\right)^{2}}-\frac{x}{2(-a d+b c)\left(b x^{2}+a\right)\left(d x^{2}+c\right)^{2}}-\frac{d(a d+11 b c) x}{8 c(-a d+b c)^{3}\left(d x^{2}+c\right)}+\frac{b^{3 / 2}(5 a d+b c) \arctan \left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2(-a d+b c)^{4} \sqrt{a}} \\
& \quad-\frac{\left(-a^{2} d^{2}+10 a c b d+15 b^{2} c^{2}\right) \arctan \left(\frac{x \sqrt{d}}{\sqrt{c}}\right) \sqrt{d}}{8 c^{3 / 2}(-a d+b c)^{4}}
\end{aligned}
$$

Result(type 3, 390 leaves):

$$
\begin{aligned}
& \frac{b^{2} x a d}{2(a d-b c)^{4}\left(b x^{2}+a\right)}-\frac{b^{3} x c}{2(a d-b c)^{4}\left(b x^{2}+a\right)}+\frac{5 b^{2} \arctan \left(\frac{b x}{\sqrt{a b}}\right) a d}{2(a d-b c)^{4} \sqrt{a b}}+\frac{b^{3} \arctan \left(\frac{b x}{\sqrt{a b}}\right) c}{2(a d-b c)^{4} \sqrt{a b}}+\frac{d^{4}}{8(a d-b c)^{4}\left(d x^{2}+c\right)^{2} c} \\
& \quad+\frac{3 d^{3} x^{3} a b}{4(a d-b c)^{4}\left(d x^{2}+c\right)^{2}}-\frac{7 d^{2} x^{3} b^{2} c}{8(a d-b c)^{4}\left(d x^{2}+c\right)^{2}}+\frac{5 d^{2} a b c x}{4(a d-b c)^{4}\left(d x^{2}+c\right)^{2}}-\frac{9 d b^{2} c^{2} x}{8(a d-b c)^{4}\left(d x^{2}+c\right)^{2}}-\frac{d^{3} a^{2} x}{8(a d-b c)^{4}\left(d x^{2}+c\right)^{2}} \\
& \quad+\frac{d^{3} \arctan \left(\frac{x d}{\sqrt{c d}}\right) a^{2}}{8(a d-b c)^{4} c \sqrt{c d}}-\frac{5 d^{2} \arctan \left(\frac{x d}{\sqrt{c d}}\right) a b}{4(a d-b c)^{4} \sqrt{c d}}-\frac{15 d c \arctan \left(\frac{x d}{\sqrt{c d}}\right) b^{2}}{8(a d-b c)^{4} \sqrt{c d}}
\end{aligned}
$$

Problem 85: Result more than twice size of optimal antiderivative.

$$
\int x^{m}\left(b x^{2}+a\right)^{2}\left(d x^{2}+c\right) \mathrm{d} x
$$

Optimal(type 3, 71 leaves, 2 steps):

$$
\frac{a^{2} c x^{1+m}}{1+m}+\frac{a(a d+2 b c) x^{3+m}}{3+m}+\frac{b(2 a d+b c) x^{5+m}}{5+m}+\frac{b^{2} d x^{7+m}}{7+m}
$$

Result(type 3, 261 leaves):
$\frac{1}{(7+m)(5+m)(3+m)(1+m)}\left(x^{1+m}\left(b^{2} d m^{3} x^{6}+9 b^{2} d m^{2} x^{6}+2 a b d m^{3} x^{4}+b^{2} c m^{3} x^{4}+23 b^{2} d m x^{6}+22 a b d m^{2} x^{4}+11 b^{2} c m^{2} x^{4}+15 b^{2} d x^{6}\right.\right.$
$+a^{2} d m^{3} x^{2}+2 a b c m^{3} x^{2}+62 a b d m x^{4}+31 b^{2} c m x^{4}+13 a^{2} d m^{2} x^{2}+26 a b c m^{2} x^{2}+42 a b d x^{4}+21 b^{2} c x^{4}+a^{2} c m^{3}+47 a^{2} d m x^{2}+94 a b c m x^{2}$
$\left.\left.+15 a^{2} c m^{2}+35 a^{2} d x^{2}+70 a b c x^{2}+71 a^{2} c m+105 a^{2} c\right)\right)$

Problem 86: Unable to integrate problem.

$$
\int \frac{x^{m}\left(b x^{2}+a\right)^{2}}{\left(d x^{2}+c\right)^{3}} \mathrm{~d} x
$$

Optimal(type 5, 163 leaves, 3 steps):

$$
\begin{aligned}
& \frac{(-a d+b c)^{2} x^{1+m}}{4 c d^{2}\left(d x^{2}+c\right)^{2}}-\frac{(-a d+b c)(a d(3-m)+b c(5+m)) x^{1+m}}{8 c^{2} d^{2}\left(d x^{2}+c\right)} \\
& \quad+\frac{\left(2 a b c d\left(-m^{2}+1\right)+a^{2} d^{2}\left(m^{2}-4 m+3\right)+b^{2} c^{2}\left(m^{2}+4 m+3\right)\right) x^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{d x^{2}}{c}\right)}{8 c^{3} d^{2}(1+m)}
\end{aligned}
$$

Result(type 8, 24 leaves):

$$
\int \frac{x^{m}\left(b x^{2}+a\right)^{2}}{\left(d x^{2}+c\right)^{3}} \mathrm{~d} x
$$

Problem 87: Unable to integrate problem.

$$
\int \frac{x^{m}\left(d x^{2}+c\right)^{2}}{b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 5, 92 leaves, 3 steps):

$$
\frac{d(-a d+2 b c) x^{1+m}}{b^{2}(1+m)}+\frac{d^{2} x^{3+m}}{b(3+m)}+\frac{(-a d+b c)^{2} x^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{a b^{2}(1+m)}
$$

Result (type 8, 24 leaves):

$$
\int \frac{x^{m}\left(d x^{2}+c\right)^{2}}{b x^{2}+a} \mathrm{~d} x
$$

Problem 88: Unable to integrate problem.

$$
\int \frac{x^{m}\left(d x^{2}+c\right)}{b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 5, 64 leaves, 2 steps):

$$
\frac{d x^{1+m}}{b(1+m)}+\frac{(-a d+b c) x^{1+m} \text { hypergeom }\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{a b(1+m)}
$$

Result(type 8, 22 leaves):

$$
\int \frac{x^{m}\left(d x^{2}+c\right)}{b x^{2}+a} \mathrm{~d} x
$$

Problem 89: Unable to integrate problem.

$$
\int \frac{x^{m}}{\left(b x^{2}+a\right)^{2}\left(d x^{2}+c\right)} \mathrm{d} x
$$

Optimal(type 5, 148 leaves, 5 steps):

$$
\begin{aligned}
& \frac{b x^{1+m}}{2 a(-a d+b c)\left(b x^{2}+a\right)}+\frac{b(b c(1-m)-a d(3-m)) x^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{2 a^{2}(-a d+b c)^{2}(1+m)} \\
& +\frac{d^{2} x^{1+m} \text { hypergeom }\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{d x^{2}}{c}\right)}{c(-a d+b c)^{2}(1+m)}
\end{aligned}
$$

Result(type 8, 24 leaves):

$$
\int \frac{x^{m}}{\left(b x^{2}+a\right)^{2}\left(d x^{2}+c\right)} \mathrm{d} x
$$

Problem 90: Unable to integrate problem.

$$
\int \frac{x^{m}\left(d x^{2}+c\right)^{3}}{\left(b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Optimal(type 5, 191 leaves, 4 steps):

$$
\begin{aligned}
& -\frac{d\left(2 b^{2} c^{2}(1+m)-3 a b c d(3+m)+a^{2} d^{2}(5+m)\right) x^{1+m}}{2 a b^{3}(1+m)}-\frac{d^{2}(b c(3+m)-a d(5+m)) x^{3+m}}{2 a b^{2}(3+m)}+\frac{(-a d+b c) x^{1+m}\left(d x^{2}+c\right)^{2}}{2 a b\left(b x^{2}+a\right)} \\
& +\frac{(-a d+b c)^{2}(a d(5+m)+b(-c m+c)) x^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{2 a^{2} b^{3}(1+m)}
\end{aligned}
$$

Result(type 8, 24 leaves):

$$
\int \frac{x^{m}\left(d x^{2}+c\right)^{3}}{\left(b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Problem 91: Unable to integrate problem.

$$
\int \frac{x^{m}\left(d x^{2}+c\right)}{\left(b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Optimal(type 5, 87 leaves, 2 steps):

$$
\frac{(-a d+b c) x^{1+m}}{2 a b\left(b x^{2}+a\right)}+\frac{(a d(1+m)+b(-c m+c)) x^{1+m} \text { hypergeom }\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{2 a^{2} b(1+m)}
$$

Result(type 8, 22 leaves):

$$
\int \frac{x^{m}\left(d x^{2}+c\right)}{\left(b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Problem 110: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{7 / 2}\left(b x^{2}+a\right)^{2}}{d x^{2}+c} \mathrm{~d} x
$$

Optimal(type 3, 230 leaves, 14 steps):

$$
\begin{aligned}
& \frac{2(-a d+b c)^{2} x^{5 / 2}}{5 d^{3}}-\frac{2 b(-2 a d+b c) x^{9 / 2}}{9 d^{2}}+\frac{2 b^{2} x^{13 / 2}}{13 d}-\frac{c^{5 / 4}(-a d+b c)^{2} \arctan \left(1-\frac{d^{1 / 4} \sqrt{2} \sqrt{x}}{c^{1 / 4}}\right) \sqrt{2}}{2 d^{17 / 4}} \\
& +\frac{c^{5 / 4}(-a d+b c)^{2} \arctan \left(1+\frac{d^{1 / 4} \sqrt{2} \sqrt{x}}{c^{1 / 4}}\right) \sqrt{2}}{2 d^{17 / 4}}-\frac{c^{5 / 4}(-a d+b c)^{2} \ln \left(\sqrt{c}+x \sqrt{d}-c^{1 / 4} d^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4 d^{17 / 4}} \\
& +\frac{c^{5 / 4}(-a d+b c)^{2} \ln \left(\sqrt{c}+x \sqrt{d}+c^{1 / 4} d^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4 d^{17 / 4}}-\frac{2 c(-a d+b c)^{2} \sqrt{x}}{d^{4}}
\end{aligned}
$$

Result(type 3, 544 leaves):

$$
\begin{aligned}
& \frac{2 b^{2} x^{13 / 2}}{13 d}+\frac{4 x^{9 / 2} a b}{9 d}-\frac{2 x^{9 / 2} b^{2} c}{9 d^{2}}+\frac{2 x^{5 / 2} a^{2}}{5 d}-\frac{4 x^{5 / 2} a b c}{5 d^{2}}+\frac{2 x^{5} / 2 b^{2} c^{2}}{5 d^{3}}-\frac{2 a^{2} c \sqrt{x}}{d^{2}}+\frac{4 a b c^{2} \sqrt{x}}{d^{3}}-\frac{2 b^{2} c^{3} \sqrt{x}}{d^{4}} \\
& +\frac{c\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{c}{d}\right)^{1 / 4}}+1\right) a^{2}}{2 d^{2}}-\frac{c^{2}\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{c}{d}\right)^{1 / 4}}+1\right) a b}{c^{3}\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{c}{d}\right)^{1 / 4}}+1\right) b^{2}} \\
& +\frac{c\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{c}{d}\right)^{1 / 4}}-1\right) a^{2}}{2 d^{2}}-\frac{c^{2}\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left.\left(\frac{c}{d}\right)^{1 / 4}-1\right) a b}\right)}{d^{3}}+\frac{c^{3}\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left.\left(\frac{c}{d}\right)^{1 / 4}-1\right) b^{2}}\right.}{2 d^{4}}
\end{aligned}
$$

$$
\begin{aligned}
& \left.\quad+\frac{c\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}\right) a^{2}}{4 d^{2}}-\frac{c^{2}\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}\right) a b}{2 d^{3}}\right) \\
& +\frac{c^{3}\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}\right) b^{2}}{4 d^{4}}
\end{aligned}
$$

Problem 111: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{3 / 2}\left(b x^{2}+a\right)^{2}}{d x^{2}+c} \mathrm{~d} x
$$

Optimal(type 3, 211 leaves, 13 steps):
$-\frac{2 b(-2 a d+b c) x^{5 / 2}}{5 d^{2}}+\frac{2 b^{2} x^{9 / 2}}{9 d}+\frac{c^{1 / 4}(-a d+b c)^{2} \arctan \left(1-\frac{d^{1 / 4} \sqrt{2} \sqrt{x}}{c^{1 / 4}}\right) \sqrt{2}}{2 d^{13 / 4}}-\frac{c^{1 / 4}(-a d+b c)^{2} \arctan \left(1+\frac{d^{1 / 4} \sqrt{2} \sqrt{x}}{c^{1 / 4}}\right) \sqrt{2}}{2 d^{13 / 4}}$
$+\frac{c^{1 / 4}(-a d+b c)^{2} \ln \left(\sqrt{c}+x \sqrt{d}-c^{1 / 4} d^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4 d^{13 / 4}}-\frac{c^{1 / 4}(-a d+b c)^{2} \ln \left(\sqrt{c}+x \sqrt{d}+c^{1 / 4} d^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4 d^{13 / 4}}$

$$
+\frac{2(-a d+b c)^{2} \sqrt{x}}{d^{3}}
$$

Result(type 3, 494 leaves):
$\frac{2 b^{2} x^{9 / 2}}{9 d}+\frac{4 x^{5 / 2} a b}{5 d}-\frac{2 x^{5 / 2} b^{2} c}{5 d^{2}}+\frac{2 a^{2} \sqrt{x}}{d}-\frac{4 a b c \sqrt{x}}{d^{2}}+\frac{2 b^{2} c^{2} \sqrt{x}}{d^{3}}-$

$+\frac{\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{c}{d}\right)^{1 / 4}}+1\right) a b c}{d^{2}}-\frac{\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{c}{d}\right)^{1 / 4}}+1\right) b^{2} c^{2}}{2 d^{3}}-\frac{\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{c}{d}\right)^{1 / 4}-1}\right) a^{2}}{2 d}$
$+\frac{\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{c}{d}\right)^{1 / 4}}-1\right) a b c}{d^{2}}-\frac{\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{c}{d}\right)^{1 / 4}}-1\right) b^{2} c^{2}}{2 d^{3}}$

$$
\begin{aligned}
& -\frac{\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}\right) a^{2}}{4 d}+\frac{\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}\right) a b c}{2 d^{2}} \\
& -\frac{\left(\frac{c}{d}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}\right) b^{2} c^{2}}{4 d^{3}}
\end{aligned}
$$

Problem 112: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(b x^{2}+a\right)^{2}}{x^{7 / 2}\left(d x^{2}+c\right)} d x
$$

Optimal(type 3, 194 leaves, 12 steps):

$$
\begin{aligned}
& -\frac{2 a^{2}}{5 c x^{5 / 2}}-\frac{(-a d+b c)^{2} \arctan \left(1-\frac{d^{1 / 4} \sqrt{2} \sqrt{x}}{c^{1 / 4}}\right) \sqrt{2}}{2 c^{9 / 4} d^{3 / 4}}+\frac{(-a d+b c)^{2} \arctan \left(1+\frac{d^{1 / 4} \sqrt{2} \sqrt{x}}{c^{1 / 4}}\right) \sqrt{2}}{2 c^{9 / 4} d^{3 / 4}} \\
& \quad+\frac{(-a d+b c)^{2} \ln \left(\sqrt{c}+x \sqrt{d}-c^{1 / 4} d^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4 c^{9 / 4} d^{3 / 4}}-\frac{(-a d+b c)^{2} \ln \left(\sqrt{c}+x \sqrt{d}+c^{1 / 4} d^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4 c^{9 / 4} d^{3 / 4}}-\frac{2 a(-a d+2 b c)}{c^{2} \sqrt{x}}
\end{aligned}
$$

Result(type 3, 451 leaves):

$$
\begin{aligned}
& \frac{\left.d \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{c}{d}\right)^{1 / 4}}+1\right) a^{2}\right)}{2 c^{2}\left(\frac{c}{d}\right)^{1 / 4}}-\frac{\sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left.\left(\frac{c}{d}\right)^{1 / 4}+1\right) a b}\right)}{c\left(\frac{c}{d}\right)^{1 / 4}}+\frac{\sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{c}{d}\right)^{1 / 4}+1}\right) b^{2}}{2 d\left(\frac{c}{d}\right)^{1 / 4}}+\frac{d \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{c}{d}\right)^{1 / 4}-1}\right) a^{2}}{2 c^{2}\left(\frac{c}{d}\right)^{1 / 4}} \\
& \quad-\frac{\sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left.\left(\frac{c}{d}\right)^{1 / 4}-1\right) a b}\right)}{c\left(\frac{c}{d}\right)^{1 / 4}}+\frac{\sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left.\left(\frac{c}{d}\right)^{1 / 4}-1\right) b^{2}}\right) d \sqrt{2} \ln \left(\frac{x-\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}{\left.x+\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}\right)}\right) a^{2}}{4 c^{2}\left(\frac{c}{d}\right)^{1 / 4}}
\end{aligned}
$$

$$
-\frac{\sqrt{2} \ln \left(\frac{x-\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}\right) a b}{2 c\left(\frac{c}{d}\right)^{1 / 4}}+\frac{\sqrt{2} \ln \left(\frac{x-\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}}{\left.x+\left(\frac{c}{d}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{c}{d}}\right)} b^{2}\right.}{4 d\left(\frac{c}{d}\right)^{1 / 4}}-\frac{2 a^{2}}{5 c x^{5 / 2}}+\frac{2 a^{2} d}{c^{2} \sqrt{x}}-\frac{4 a b}{c \sqrt{x}}
$$

Problem 120: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(d x^{2}+c\right)^{3}}{\left(b x^{2}+a\right) \sqrt{x}} \mathrm{~d} x
$$

Optimal(type 3, 227 leaves, 12 steps):

$$
\begin{aligned}
& \frac{2 d^{2}(-a d+3 b c) x^{5 / 2}}{5 b^{2}}+\frac{2 d^{3} x^{9 / 2}}{9 b}-\frac{(-a d+b c)^{3} \arctan \left(1-\frac{b^{1 / 4} \sqrt{2} \sqrt{x}}{a^{1 / 4}}\right) \sqrt{2}}{2 a^{3 / 4} b^{13 / 4}}+\frac{(-a d+b c)^{3} \arctan \left(1+\frac{b^{1 / 4} \sqrt{2} \sqrt{x}}{a^{1 / 4}}\right) \sqrt{2}}{2 a^{3 / 4} b^{13 / 4}} \\
& \quad-\frac{(-a d+b c)^{3} \ln \left(\sqrt{a}+x \sqrt{b}-a^{1 / 4} b^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4 a^{3 / 4} b^{13 / 4}}+\frac{(-a d+b c)^{3} \ln \left(\sqrt{a}+x \sqrt{b}+a^{1 / 4} b^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4 a^{3 / 4} b^{13 / 4}} \\
& \quad+\frac{2 d\left(a^{2} d^{2}-3 a c b d+3 b^{2} c^{2}\right) \sqrt{x}}{b^{3}}
\end{aligned}
$$

Result(type 3, 649 leaves):

$$
\begin{aligned}
& \frac{2 d^{3} x^{9 / 2}}{9 b}-\frac{2 d^{3} x^{5 / 2} a}{5 b^{2}}+\frac{6 d^{2} x^{5} / 2 c}{5 b}+\frac{2 d^{3} a^{2} \sqrt{x}}{b^{3}}-\frac{6 d^{2} a c \sqrt{x}}{b^{2}}+\frac{6 d c^{2} \sqrt{x}}{b}-\frac{\left(\frac{a}{b}\right)^{1 / 4} a^{2} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}+1}\right) d^{3}}{2 b^{3}} \\
& +\frac{\left.3\left(\frac{a}{b}\right)^{1 / 4} a \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}+1\right) c d^{2}\right) 3\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left.\left(\frac{a}{b}\right)^{1 / 4}+1\right) c^{2} d}\right)}{2 b^{2}}-\frac{\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}+1}\right) c^{3}}{2 a} \\
& -\frac{\left(\frac{a}{b}\right)^{1 / 4} a^{2} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left.\left(\frac{a}{b}\right)^{1 / 4}-1\right) d^{3}}\right)}{2 b^{3}}+\frac{3\left(\frac{a}{b}\right)^{1 / 4} a \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left.\left(\frac{a}{b}\right)^{1 / 4}-1\right) c d^{2}}\right)}{2 b^{2}}-\frac{3\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left.\left(\frac{a}{b}\right)^{1 / 4}-1\right) c^{2} d}\right.}{2 b}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{\left.\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}-1\right) c^{3}\right)}{2 a}-\frac{\left(\frac{a}{b}\right)^{1 / 4} a^{2} \sqrt{2} \ln \left(\frac{x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}\right) d^{3}}{4 b^{3}} \\
& +\frac{3\left(\frac{a}{b}\right)^{1 / 4} a \sqrt{2} \ln \left(\frac{x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}\right) c d^{2}}{4 b^{2}}-\frac{3\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}\right) c^{2} d}{4 b} \\
& +\frac{\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}\right) c^{3}}{4 a}
\end{aligned}
$$

Problem 121: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(d x^{2}+c\right)^{3}}{x^{7 / 2}\left(b x^{2}+a\right)} \mathrm{d} x
$$

Optimal(type 3, 206 leaves, 12 steps):

$$
\begin{aligned}
-\frac{2 c^{3}}{5 a x^{5 / 2}}+\frac{2 d^{3} x^{3 / 2}}{3 b}-\frac{(-a d+b c)^{3} \arctan \left(1-\frac{b^{1 / 4} \sqrt{2} \sqrt{x}}{a^{1 / 4}}\right) \sqrt{2}}{2 a^{9 / 4} b^{7 / 4}}+\frac{(-a d+b c)^{3} \arctan \left(1+\frac{b^{1 / 4} \sqrt{2} \sqrt{x}}{a^{1 / 4}}\right) \sqrt{2}}{2 a^{9 / 4} b^{7 / 4}} \\
\quad+\frac{(-a d+b c)^{3} \ln \left(\sqrt{a}+x \sqrt{b}-a^{1 / 4} b^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4 a^{9 / 4} b^{7 / 4}}-\frac{(-a d+b c)^{3} \ln \left(\sqrt{a}+x \sqrt{b}+a^{1 / 4} b^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{4 a^{9 / 4} b^{7 / 4}}+\frac{2 c^{2}(-3 a d+b c)}{a^{2} \sqrt{x}}
\end{aligned}
$$

Result(type 3, 615 leaves):


Problem 122: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{7 / 2}\left(d x^{2}+c\right)^{3}}{\left(b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 317 leaves, 13 steps):
$\frac{d\left(17 a^{2} d^{2}-39 a c b d+27 b^{2} c^{2}\right) x^{5 / 2}}{10 b^{4}}+\frac{d^{2}(-17 a d+39 b c) x^{9 / 2}}{18 b^{3}}+\frac{17 d^{3} x^{13 / 2}}{26 b^{2}}-\frac{x^{5 / 2}\left(d x^{2}+c\right)^{3}}{2 b\left(b x^{2}+a\right)}$

$$
+\frac{a^{1 / 4}(-17 a d+5 b c)(-a d+b c)^{2} \arctan \left(1-\frac{b^{1 / 4} \sqrt{2} \sqrt{x}}{a^{1 / 4}}\right) \sqrt{2}}{8 b^{21 / 4}}-\frac{a^{1 / 4}(-17 a d+5 b c)(-a d+b c)^{2} \arctan \left(1+\frac{b^{1 / 4} \sqrt{2} \sqrt{x}}{a^{1 / 4}}\right) \sqrt{2}}{8 b^{21 / 4}}
$$

$$
+\frac{a^{1 / 4}(-17 a d+5 b c)(-a d+b c)^{2} \ln \left(\sqrt{a}+x \sqrt{b}-a^{1 / 4} b^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{16 b^{21 / 4}}
$$

$$
-\frac{a^{1 / 4}(-17 a d+5 b c)(-a d+b c)^{2} \ln \left(\sqrt{a}+x \sqrt{b}+a^{1 / 4} b^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{16 b^{21 / 4}}+\frac{(-17 a d+5 b c)(-a d+b c)^{2} \sqrt{x}}{2 b^{5}}
$$

Result(type 3, 803 leaves):
$-\frac{4 x^{9 / 2} a d^{3}}{9 b^{3}}+\frac{2 x^{9 / 2} c d^{2}}{3 b^{2}}+\frac{6 x^{5 / 2} a^{2} d^{3}}{5 b^{4}}+\frac{6 x^{5 / 2} c^{2} d}{5 b^{2}}-\frac{8 a^{3} d^{3} \sqrt{x}}{b^{5}}-$

$$
\frac{39 a^{2}\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}+1\right) c d^{2}}{8 b^{4}}
$$

$$
\begin{aligned}
& +\frac{3 \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left.\left(\frac{a}{b}\right)^{1 / 4}+1\right) c d^{2}}\right)}{2 b\left(\frac{a}{b}\right)^{1 / 4}}-\frac{3 \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}+1}\right) c^{2} d}{2 a\left(\frac{a}{b}\right)^{1 / 4}}+\frac{b \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left.\left(\frac{a}{b}\right)^{1 / 4}+1\right) c^{3}}\right)}{2 a^{2}\left(\frac{a}{b}\right)^{1 / 4}}-\frac{a \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}-1} d^{3}\right.}{2 b^{2}\left(\frac{a}{b}\right)^{1 / 4}} \\
& +\frac{3 \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left.\left(\frac{a}{b}\right)^{1 / 4}-1\right) c d^{2}}\right)}{2 b\left(\frac{a}{b}\right)^{1 / 4}}-\frac{3 \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}-1}\right) c^{2} d}{2 a\left(\frac{a}{b}\right)^{1 / 4}}+\frac{b \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}-1}\right) c^{3}}{2 a^{2}\left(\frac{a}{b}\right)^{1 / 4}}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{27 a\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}+1\right) c^{2} d \quad 39 a^{2}\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}-1}\right) c d^{2}}{8 b^{3}}-\frac{8 b^{4}}{} \\
& +\frac{27 a\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}-1\right) c^{2} d \quad 39 a^{2}\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}\right) c d^{2}}{16 b^{3}} \\
& +\frac{27 a\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}\right) c^{2} d}{16 b^{3}}+\frac{2 d^{3} x^{13 / 2}}{13 b^{2}}+\frac{2 c^{3} \sqrt{x}}{b^{2}}-\frac{12 a c^{2} d \sqrt{x}}{b^{3}}-\frac{a^{4} \sqrt{x} d^{3}}{2 b^{5}\left(b x^{2}+a\right)}+\frac{a \sqrt{x} c^{3}}{2 b^{2}\left(b x^{2}+a\right)} \\
& -\frac{\left.5\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}+1\right) c^{3}\right)}{8 b^{2}}-\frac{5\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}-1\right) c^{3}}{8 b^{2}} \\
& -\frac{5\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}\right) c^{3}}{16 b^{2}}-\frac{12 x^{5} / 2 a c d^{2}}{5 b^{3}}+\frac{18 a^{2} c d^{2} \sqrt{x}}{b^{4}}+\frac{3 a^{3} \sqrt{x} c d^{2}}{2 b^{4}\left(b x^{2}+a\right)}-\frac{3 a^{2} \sqrt{x} c^{2} d}{2 b^{3}\left(b x^{2}+a\right)} \\
& +\frac{\left.17 a^{3}\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}+1\right) d^{3}\right)}{8 b^{5}}+\frac{17 a^{3}\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}-1\right) d^{3}}{8 b^{5}} \\
& +\frac{17 a^{3}\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}\right) d^{3}}{16 b^{5}}
\end{aligned}
$$

[^1]$$
\int \frac{x^{5 / 2}\left(d x^{2}+c\right)^{3}}{\left(b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 286 leaves, 13 steps):

$$
\begin{aligned}
& \frac{d\left(5 a^{2} d^{2}-11 a c b d+7 b^{2} c^{2}\right) x^{3 / 2}}{2 b^{4}}+\frac{3 d^{2}(-5 a d+11 b c) x^{7 / 2}}{14 b^{3}}+\frac{15 d^{3} x^{11 / 2}}{22 b^{2}}-\frac{x^{3 / 2}\left(d x^{2}+c\right)^{3}}{2 b\left(b x^{2}+a\right)} \\
& -\frac{3(-5 a d+b c)(-a d+b c)^{2} \arctan \left(1-\frac{b^{1 / 4} \sqrt{2} \sqrt{x}}{a^{1 / 4}}\right) \sqrt{2}}{8 a^{1 / 4} b^{19 / 4}}+\frac{3(-5 a d+b c)(-a d+b c)^{2} \arctan \left(1+\frac{b^{1 / 4} \sqrt{2} \sqrt{x}}{a^{1 / 4}}\right) \sqrt{2}}{8 a^{1 / 4} b^{19 / 4}} \\
& +\frac{3(-5 a d+b c)(-a d+b c)^{2} \ln \left(\sqrt{a}+x \sqrt{b}-a^{1 / 4} b^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{16 a^{1 / 4} b^{19 / 4}}-\frac{3(-5 a d+b c)(-a d+b c)^{2} \ln \left(\sqrt{a}+x \sqrt{b}+a^{1 / 4} b^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{16 a^{1 / 4} b^{19 / 4}}
\end{aligned}
$$

Result(type 3, 747 leaves):

$$
\begin{aligned}
& \frac{2 d^{3} x^{11 / 2}}{11 b^{2}}-\frac{4 d^{3} x^{7 / 2} a}{7 b^{3}}+\frac{6 d^{2} c x^{7 / 2}}{7 b^{2}}+\frac{2 d^{3} x^{3 / 2} a^{2}}{b^{4}}-\frac{4 d^{2} x^{3 / 2} a c}{b^{3}}+\frac{2 d x^{3 / 2} c^{2}}{b^{2}}+\frac{x^{3 / 2} a^{3} d^{3}}{2 b^{4}\left(b x^{2}+a\right)}-\frac{3 x^{3 / 2} a^{2} c d^{2}}{2 b^{3}\left(b x^{2}+a\right)}+\frac{3 x^{3 / 2} a c^{2} d}{2 b^{2}\left(b x^{2}+a\right)} \\
& -\frac{x^{3 / 2} c^{3}}{2 b\left(b x^{2}+a\right)}-\frac{15 \sqrt{2} \ln \left(\frac{x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{\left.x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}\right) a^{3} d^{3}}\right.}{16 b^{5}\left(\frac{a}{b}\right)^{1 / 4}}-\frac{15 \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}+1\right) a^{3} d^{3}}{8 b^{5}\left(\frac{a}{b}\right)^{1 / 4}}-\frac{15 \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}-1}\right)^{a^{3} d^{3}}}{8 b^{5}\left(\frac{a}{b}\right)^{1 / 4}} \\
& +\frac{33 \sqrt{2} \ln \left(\frac{x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}\right) a^{2} c d^{2}}{16 b^{4}\left(\frac{a}{b}\right)^{1 / 4}}+\frac{33 \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}+1\right) a^{2} c d^{2}}{8 b^{4}\left(\frac{a}{b}\right)^{1 / 4}}+\frac{33 \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}-1}\right)^{2} c d^{2}}{8 b^{4}\left(\frac{a}{b}\right)^{1 / 4}} \\
& -\frac{21 \sqrt{2} \ln \left(\frac{x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}\right) a c^{2} d}{16 b^{3}\left(\frac{a}{b}\right)^{1 / 4}}-\frac{21 \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}+1}\right) a c^{2} d}{8 b^{3}\left(\frac{a}{b}\right)^{1 / 4}}-\frac{21 \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}-1}\right) a c^{2} d}{8 b^{3}\left(\frac{a}{b}\right)^{1 / 4}}
\end{aligned}
$$

$$
+\frac{3 \sqrt{2} \ln \left(\frac{x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}\right) c^{3}}{16 b^{2}\left(\frac{a}{b}\right)^{1 / 4}}+\frac{\left.3 \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}+1\right) c^{3}\right)}{8 b^{2}\left(\frac{a}{b}\right)^{1 / 4}}+\frac{3 \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}-1\right) c^{3}}{8 b^{2}\left(\frac{a}{b}\right)^{1 / 4}}
$$

Problem 124: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(d x^{2}+c\right)^{3}}{x^{9 / 2}\left(b x^{2}+a\right)^{2}} d x
$$

Optimal(type 3, 292 leaves, 13 steps):
$-\frac{c^{2}(-7 a d+11 b c)}{14 a^{2} b x^{7 / 2}}+\frac{c\left(6 a^{2} d^{2}-21 a c b d+11 b^{2} c^{2}\right)}{6 a^{3} b x^{3 / 2}}+\frac{(-a d+b c)\left(d x^{2}+c\right)^{2}}{2 a b x^{7 / 2}\left(b x^{2}+a\right)}-\frac{(-a d+b c)^{2}(a d+11 b c) \arctan \left(1-\frac{b^{1 / 4} \sqrt{2} \sqrt{x}}{a^{1 / 4}}\right) \sqrt{2}}{8 a^{15 / 4} b^{5 / 4}}$
$+\frac{(-a d+b c)^{2}(a d+11 b c) \arctan \left(1+\frac{b^{1 / 4} \sqrt{2} \sqrt{x}}{a^{1 / 4}}\right) \sqrt{2}}{8 a^{15 / 4} b^{5 / 4}}-\frac{(-a d+b c)^{2}(a d+11 b c) \ln \left(\sqrt{a}+x \sqrt{b}-a^{1 / 4} b^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{16 a^{15 / 4} b^{5 / 4}}$
$+\frac{(-a d+b c)^{2}(a d+11 b c) \ln \left(\sqrt{a}+x \sqrt{b}+a^{1 / 4} b^{1 / 4} \sqrt{2} \sqrt{x}\right) \sqrt{2}}{16 a^{15 / 4} b^{5 / 4}}$
Result(type 3, 705 leaves):

$$
\frac{\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}+1\right) d^{3}}{8 a b}
$$

$$
\begin{aligned}
& -\frac{2 c^{3}}{7 a^{2} x^{7 / 2}}-\frac{2 c^{2} d}{a^{2} x^{3 / 2}}+\frac{4 c^{3} b}{3 a^{3} x^{3 / 2}}-\frac{\sqrt{x} d^{3}}{2 b\left(b x^{2}+a\right)}+\frac{3 \sqrt{x} c d^{2}}{2 a\left(b x^{2}+a\right)}-\frac{3 b \sqrt{x} c^{2} d}{2 a^{2}\left(b x^{2}+a\right)}+\frac{b^{2} \sqrt{x} c^{3}}{2 a^{3}\left(b x^{2}+a\right)}+ \\
& +\frac{9\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}+1\right) c d^{2} \quad 21 b\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left.\left(\frac{a}{b}\right)^{1 / 4}+1\right) c^{2} d}\right)}{8 a^{2}}+\frac{11 b^{2}\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}+1} a^{3}\right.}{8 a^{3}} \\
& +\frac{\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}-1\right) d^{3} \quad 9\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}-1}\right) c d^{2} \quad 21 b\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}-1}\right)^{c^{2} d}}{8 a b}+\frac{8 a^{2}}{8 a^{3}}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{\left.11 b^{2}\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{x}}{\left(\frac{a}{b}\right)^{1 / 4}}-1\right) c^{3}\right)}{8 a^{4}}+\frac{\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}\right) d^{3}}{16 a b} \\
& +\frac{9\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}\right) c d^{2} \quad 21 b\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{16 a^{2}}\right) c^{2} d}{\left.x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}\right)} 16 a^{3} \\
& +\frac{11 b^{2}\left(\frac{a}{b}\right)^{1 / 4} \sqrt{2} \ln \left(\frac{x+\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{1 / 4} \sqrt{x} \sqrt{2}+\sqrt{\frac{a}{b}}}\right) c^{3}}{16 a^{4}}
\end{aligned}
$$

Problem 160: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(b x^{2}+a\right)^{2} \sqrt{d x^{2}+c}}{x^{7}} \mathrm{~d} x
$$

Optimal(type 3, 129 leaves, 6 steps):

$$
-\frac{a^{2}\left(d x^{2}+c\right)^{3 / 2}}{6 c x^{6}}-\frac{a(-a d+4 b c)\left(d x^{2}+c\right)^{3 / 2}}{8 c^{2} x^{4}}-\frac{d\left(a^{2} d^{2}-4 a c b d+8 b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{d x^{2}+c}}{\sqrt{c}}\right)}{16 c^{5 / 2}}-\frac{\left(a^{2} d^{2}-4 a c b d+8 b^{2} c^{2}\right) \sqrt{d x^{2}+c}}{16 c^{2} x^{2}}
$$

Result(type 3, 280 leaves):

$$
\begin{aligned}
& -\frac{a^{2}\left(d x^{2}+c\right)^{3 / 2}}{6 c x^{6}}+\frac{a^{2} d\left(d x^{2}+c\right)^{3 / 2}}{8 c^{2} x^{4}}-\frac{a^{2} d^{2}\left(d x^{2}+c\right)^{3 / 2}}{16 c^{3} x^{2}}-\frac{a^{2} d^{3} \ln \left(\frac{2 c+2 \sqrt{c} \sqrt{d x^{2}+c}}{x}\right)}{16 c^{5 / 2}}+\frac{a^{2} d^{3} \sqrt{d x^{2}+c}}{16 c^{3}}-\frac{b^{2}\left(d x^{2}+c\right)^{3 / 2}}{2 c x^{2}} \\
& -\frac{b^{2} d \ln \left(\frac{2 c+2 \sqrt{c} \sqrt{d x^{2}+c}}{x}\right)}{2 \sqrt{c}}+\frac{b^{2} d \sqrt{d x^{2}+c}}{2 c}-\frac{a b\left(d x^{2}+c\right)^{3 / 2}}{2 c x^{4}}+\frac{a b d\left(d x^{2}+c\right)^{3 / 2}}{4 c^{2} x^{2}}+\frac{a b d^{2} \ln \left(\frac{2 c+2 \sqrt{c} \sqrt{d x^{2}+c}}{x}\right)}{4 c^{3 / 2}} \\
& \quad-\frac{a b d^{2} \sqrt{d x^{2}+c}}{4 c^{2}}
\end{aligned}
$$

[^2]$$
\int \frac{\left(b x^{2}+a\right)^{2}\left(d x^{2}+c\right)^{3 / 2}}{x^{7}} \mathrm{~d} x
$$

Optimal(type 3, 163 leaves, 7 steps):

$$
\begin{aligned}
& -\frac{\left(24 b^{2} c^{2}+a d(-a d+12 b c)\right)\left(d x^{2}+c\right)^{3 / 2}}{48 c^{2} x^{2}}-\frac{a^{2}\left(d x^{2}+c\right)^{5 / 2}}{6 c x^{6}}-\frac{a(-a d+12 b c)\left(d x^{2}+c\right)^{5 / 2}}{24 c^{2} x^{4}} \\
& -\frac{d\left(24 b^{2} c^{2}+a d(-a d+12 b c)\right) \operatorname{arctanh}\left(\frac{\sqrt{d x^{2}+c}}{\sqrt{c}}\right)}{16 c^{3 / 2}}+\frac{d\left(24 b^{2} c^{2}+a d(-a d+12 b c)\right) \sqrt{d x^{2}+c}}{16 c^{2}}
\end{aligned}
$$

Result(type 3, 334 leaves):

$$
\begin{aligned}
& -\frac{a^{2}\left(d x^{2}+c\right)^{5 / 2}}{6 c x^{6}}+\frac{a^{2} d\left(d x^{2}+c\right)^{5 / 2}}{24 c^{2} x^{4}}+\frac{a^{2} d^{2}\left(d x^{2}+c\right)^{5 / 2}}{48 c^{3} x^{2}}-\frac{a^{2} d^{3}\left(d x^{2}+c\right)^{3 / 2}}{48 c^{3}}+\frac{a^{2} d^{3} \ln \left(\frac{2 c+2 \sqrt{c} \sqrt{d x^{2}+c}}{x}\right)}{16 c^{3 / 2}}-\frac{a^{2} d^{3} \sqrt{d x^{2}+c}}{16 c^{2}} \\
& -\frac{b^{2}\left(d x^{2}+c\right)^{5 / 2}}{2 c x^{2}}+\frac{b^{2} d\left(d x^{2}+c\right)^{3 / 2}}{2 c}-\frac{3 b^{2} d \sqrt{c} \ln \left(\frac{2 c+2 \sqrt{c} \sqrt{d x^{2}+c}}{x}\right)}{2}+\frac{3 b^{2} d \sqrt{d x^{2}+c}}{2}-\frac{a b\left(d x^{2}+c\right)^{5 / 2}}{2 c x^{4}}-\frac{a b d\left(d x^{2}+c\right)^{5 / 2}}{4 c^{2} x^{2}} \\
& +\frac{a b d^{2}\left(d x^{2}+c\right)^{3 / 2}}{4 c^{2}}-\frac{3 a b d^{2} \ln \left(\frac{2 c+2 \sqrt{c} \sqrt{d x^{2}+c}}{x}\right)}{4 \sqrt{c}}+\frac{3 a b d^{2} \sqrt{d x^{2}+c}}{4 c}
\end{aligned}
$$

Problem 184: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{d x^{2}+c}}{b x^{2}+a} d x
$$

Optimal(type 3, 65 leaves, 5 steps):

$$
\frac{\operatorname{arctanh}\left(\frac{x \sqrt{d}}{\sqrt{d x^{2}+c}}\right) \sqrt{d}}{b}+\frac{\arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right) \sqrt{-a d+b c}}{b \sqrt{a}}
$$

Result(type 3, 947 leaves):
$\frac{\sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{2 \sqrt{-a b}}$
$+\frac{\sqrt{d} \ln \left(\frac{\frac{d \sqrt{-a b}}{b}+\left(x-\frac{\sqrt{-a b}}{b}\right) d}{\sqrt{d}}+\sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right)}{2 b}$

$$
\left.+\frac{\ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right) a d}{2 \sqrt{-a b} b \sqrt{-\frac{a d-b c}{b}}}\right)
$$

$$
\left.-\frac{\ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right) c}{2 \sqrt{-a b} \sqrt{-\frac{a d-b c}{b}}}\right)
$$

$$
-\frac{\sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{2 \sqrt{-a b}}
$$

$$
+\frac{\sqrt{d} \ln \left(\frac{-\frac{d \sqrt{-a b}}{b}+\left(x+\frac{\sqrt{-a b}}{b}\right) d}{\sqrt{d}}+\sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right)}{2 b}
$$

$$
\left.-\frac{\ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right) a d}{2 \sqrt{-a b} b \sqrt{-\frac{a d-b c}{b}}}\right)
$$

$$
\ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right) c
$$

Problem 185: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{3}\left(d x^{2}+c\right)^{3 / 2}}{b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 3, 95 leaves, 6 steps):

$$
-\frac{a\left(d x^{2}+c\right)^{3 / 2}}{3 b^{2}}+\frac{\left(d x^{2}+c\right)^{5 / 2}}{5 b d}+\frac{a(-a d+b c)^{3 / 2} \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{d x^{2}+c}}{\sqrt{-a d+b c}}\right)}{b^{7 / 2}}-\frac{a(-a d+b c) \sqrt{d x^{2}+c}}{b^{3}}
$$

Result(type 3, 1896 leaves):

$$
\frac{\left(d x^{2}+c\right)^{5 / 2}}{5 b d}-\frac{a\left(d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2}}{6 b^{2}}
$$

$$
-\frac{a d \sqrt{-a b} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b} x}}{4 b^{3}}
$$

$$
-\frac{3 a \sqrt{d} \sqrt{-a b} \ln \left(\frac{\frac{d \sqrt{-a b}}{b}+\left(x-\frac{\sqrt{-a b}}{b}\right) d}{\sqrt{d}}+\sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right) c}{4 b^{3}}
$$

$$
+\frac{a^{2} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}} d}{2 b^{3}}-\frac{a \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}} c}{2 b^{2}}
$$

$+\frac{a^{2} d^{3 / 2} \sqrt{-a b} \ln \left(\frac{\frac{d \sqrt{-a b}}{b}+\left(x-\frac{\sqrt{-a b}}{b}\right) d}{\sqrt{d}}+\sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right)}{2 b^{4}}$
$+\frac{a^{3} \ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right) d^{2}}{2 b^{4} \sqrt{-\frac{a d-b c}{b}}}$
$a^{2} \ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right) d c$
$b^{3} \sqrt{-\frac{a d-b c}{b}}$
$+\frac{a \ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right) c^{2}}{2 b^{2} \sqrt{-\frac{a d-b c}{b}}}$
$-\frac{a\left(d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2}}{6 b^{2}}$
$+\frac{a d \sqrt{-a b} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b} x}}{4 b^{3}}$
$+\frac{3 a \sqrt{d} \sqrt{-a b} \ln \left(\frac{-\frac{d \sqrt{-a b}}{b}+\left(x+\frac{\sqrt{-a b}}{b}\right) d}{\sqrt{d}}+\sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right) c}{4 b^{3}}$

$$
+\frac{a^{2} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}} d}{2 b^{3}}-\frac{a \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b} c}}{2 b^{2}}
$$

$$
-\frac{a^{2} d^{3} / 2 \sqrt{-a b} \ln \left(\frac{-\frac{d \sqrt{-a b}}{b}+\left(x+\frac{\sqrt{-a b}}{b}\right) d}{\sqrt{d}}+\sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right)}{2 b^{4}}
$$

$$
+\frac{a^{3} \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right) d^{2}}{2 b^{4} \sqrt{-\frac{a d-b c}{b}}}
$$

$$
a^{2} \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right) d c
$$

$$
b^{3} \sqrt{-\frac{a d-b c}{b}}
$$

$$
+\frac{a \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right) c^{2}}{2 b^{2} \sqrt{-\frac{a d-b c}{b}}}
$$

Problem 186: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(d x^{2}+c\right)^{3 / 2}}{x^{2}\left(b x^{2}+a\right)} d x
$$

Optimal(type 3, 84 leaves, 6 steps):

$$
-\frac{(-a d+b c)^{3 / 2} \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{a^{3 / 2} b}+\frac{d^{3 / 2} \operatorname{arctanh}\left(\frac{x \sqrt{d}}{\sqrt{d x^{2}+c}}\right)}{b}-\frac{c \sqrt{d x^{2}+c}}{a x}
$$

Result(type 3, 1955 leaves):

$$
\begin{aligned}
& -\frac{\left(d x^{2}+c\right)^{5 / 2}}{a c x}+\frac{d x\left(d x^{2}+c\right)^{3 / 2}}{a c}+\frac{3 d x \sqrt{d x^{2}+c}}{2 a}+\frac{3 \sqrt{d} c \ln \left(x \sqrt{d}+\sqrt{d x^{2}+c}\right)}{2 a} \\
& \left.-\frac{b\left(d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2}}{6 a \sqrt{-a b}}-\frac{d \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b} x}}{} \begin{array}{l}
x a
\end{array}\right)
\end{aligned}
$$

$$
-\frac{3 \sqrt{d} \ln \left(\frac{\frac{d \sqrt{-a b}}{b}+\left(x-\frac{\sqrt{-a b}}{b}\right) d}{\sqrt{d}}+\sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right) c}{4 a}
$$

$$
+\frac{\sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}} d}{2 \sqrt{-a b}}-\frac{b \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b} c}}{2 a \sqrt{-a b}}
$$

$$
+\frac{d^{3} / 2 \ln \left(\frac{\frac{d \sqrt{-a b}}{b}+\left(x-\frac{\sqrt{-a b}}{b}\right) d}{\sqrt{d}}+\sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right)}{2 b}
$$

$$
+\frac{a \ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right)}{2 b \sqrt{-a b} \sqrt{-\frac{a d-b c}{b}}}
$$

$$
\begin{aligned}
& \ln \left(\frac{\left.-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right)}{x-\frac{\sqrt{-a b}}{b}}\right) d c \\
& +\frac{b \ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right) c^{2}}{2 a \sqrt{-a b} \sqrt{-\frac{a d-b c}{b}}} \\
& +\frac{b\left(d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2}}{6 a \sqrt{-a b}}-\frac{d \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b} x}}{4 a} \\
& -\frac{3 \sqrt{d} \ln \left(\frac{-\frac{d \sqrt{-a b}}{b}+\left(x+\frac{\sqrt{-a b}}{b}\right) d}{\sqrt{d}}+\sqrt{\left.d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right) c}\right.}{4 a} \\
& -\frac{\sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}} d}{2 \sqrt{-a b}}+\frac{b \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}} c}{2 a \sqrt{-a b}} \\
& +\frac{d^{3} / 2 \ln \left(\frac{-\frac{d \sqrt{-a b}}{b}+\left(x+\frac{\sqrt{-a b}}{b}\right) d}{\sqrt{d}}+\sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right)}{2 b}
\end{aligned}
$$

$$
-\frac{a \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right) d^{2}}{2 b \sqrt{-a b} \sqrt{-\frac{a d-b c}{b}}}
$$

$$
+\frac{\ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right) d c}{\sqrt{-a b} \sqrt{-\frac{a d-b c}{b}}}
$$

$$
-\frac{b \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right) c^{2}}{2 a \sqrt{-a b} \sqrt{-\frac{a d-b c}{b}}}
$$

Problem 187: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(d x^{2}+c\right)^{3 / 2}}{x^{4}\left(b x^{2}+a\right)} \mathrm{d} x
$$

Optimal(type 3, 84 leaves, 5 steps):

$$
\frac{(-a d+b c)^{3 / 2} \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{a^{5 / 2}}-\frac{c \sqrt{d x^{2}+c}}{3 a x^{3}}+\frac{(-4 a d+3 b c) \sqrt{d x^{2}+c}}{3 a^{2} x}
$$

Result(type ?, 2088 leaves): Display of huge result suppressed!
Problem 188: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{4}\left(d x^{2}+c\right)^{5 / 2}}{b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 3, 257 leaves, 9 steps):
$\frac{d x^{5}\left(d x^{2}+c\right)^{3 / 2}}{8 b}+\frac{a^{3 / 2}(-a d+b c)^{5 / 2} \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{b^{5}}$

$$
-\left(-128 a^{4} d^{4}+320 a^{3} b c d^{3}-240 a^{2} b^{2} c^{2} d^{2}+40 a b^{3} c^{3} d+5 b^{4} c^{4}\right) \operatorname{arctanh}\left(\frac{x \sqrt{d}}{\sqrt{d x^{2}+c}}\right)
$$

## $128 b^{5} d^{3 / 2}$

$$
+\frac{\left(-64 a^{3} d^{3}+144 a^{2} b c d^{2}-88 a b^{2} c^{2} d+5 b^{3} c^{3}\right) x \sqrt{d x^{2}+c}}{128 b^{4} d}+\frac{\left(48 a^{2} d^{2}-104 a c b d+59 b^{2} c^{2}\right) x^{3} \sqrt{d x^{2}+c}}{192 b^{3}}+\frac{d(-8 a d+11 b c) x^{5} \sqrt{d x^{2}+c}}{48 b^{2}}
$$

Result(type ?, 3372 leaves): Display of huge result suppressed!
Problem 189: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{3}\left(d x^{2}+c\right)^{5 / 2}}{b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 3, 120 leaves, 7 steps):

$$
-\frac{a(-a d+b c)\left(d x^{2}+c\right)^{3 / 2}}{3 b^{3}}-\frac{a\left(d x^{2}+c\right)^{5 / 2}}{5 b^{2}}+\frac{\left(d x^{2}+c\right)^{7 / 2}}{7 b d}+\frac{a(-a d+b c)^{5 / 2} \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{d x^{2}+c}}{\sqrt{-a d+b c}}\right)}{b^{9 / 2}}-\frac{a(-a d+b c)^{2} \sqrt{d x^{2}+c}}{b^{4}}
$$

Result(type ?, 3126 leaves): Display of huge result suppressed!
Problem 190: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(d x^{2}+c\right)^{5 / 2}}{b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 3, 130 leaves, 7 steps):

$$
\begin{aligned}
& \frac{d x\left(d x^{2}+c\right)^{3 / 2}}{4 b}+\frac{(-a d+b c)^{5 / 2} \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{b^{3} \sqrt{a}}+\frac{\left(8 a^{2} d^{2}-20 a c b d+15 b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{x \sqrt{d}}{\sqrt{d x^{2}+c}}\right) \sqrt{d}}{8 b^{3}} \\
& \quad+\frac{d(-4 a d+7 b c) x \sqrt{d x^{2}+c}}{8 b^{2}}
\end{aligned}
$$

Result(type ?, 3100 leaves): Display of huge result suppressed!
Problem 191: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(d x^{2}+c\right)^{5 / 2}}{x^{2}\left(b x^{2}+a\right)} \mathrm{d} x
$$

Optimal(type 3, 121 leaves, 7 steps):

$$
-\frac{c\left(d x^{2}+c\right)^{3 / 2}}{a x}-\frac{(-a d+b c)^{5 / 2} \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{a^{3 / 2} b^{2}}+\frac{d^{3 / 2}(-2 a d+5 b c) \operatorname{arctanh}\left(\frac{x \sqrt{d}}{\sqrt{d x^{2}+c}}\right)}{2 b^{2}}+\frac{d(a d+2 b c) x \sqrt{d x^{2}+c}}{2 a b}
$$

Result(type ?, 3190 leaves): Display of huge result suppressed!
Problem 192: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(d x^{2}+c\right)^{5 / 2}}{x^{4}\left(b x^{2}+a\right)} \mathrm{d} x
$$

Optimal(type 3, 108 leaves, 7 steps):

$$
-\frac{c\left(d x^{2}+c\right)^{3 / 2}}{3 a x^{3}}+\frac{(-a d+b c)^{5 / 2} \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{a^{5 / 2} b}+\frac{d^{5 / 2} \operatorname{arctanh}\left(\frac{x \sqrt{d}}{\sqrt{d x^{2}+c}}\right)}{b}+\frac{c(-2 a d+b c) \sqrt{d x^{2}+c}}{a^{2} x}
$$

Result(type ?, 3345 leaves): Display of huge result suppressed!
Problem 193: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{5}}{\left(b x^{2}+a\right) \sqrt{d x^{2}+c}} \mathrm{~d} x
$$

Optimal(type 3, 84 leaves, 5 steps):

$$
\frac{\left(d x^{2}+c\right)^{3 / 2}}{3 b d^{2}}-\frac{a^{2} \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{d x^{2}+c}}{\sqrt{-a d+b c}}\right)}{b^{5 / 2} \sqrt{-a d+b c}}-\frac{(a d+b c) \sqrt{d x^{2}+c}}{d^{2} b^{2}}
$$

Result(type 3, 361 leaves):

$$
\frac{x^{2} \sqrt{d x^{2}+c}}{3 b d}-\frac{2 c \sqrt{d x^{2}+c}}{3 b d^{2}}-\frac{a \sqrt{d x^{2}+c}}{b^{2} d}
$$

$$
a^{2} \ln \left(\frac{\left.-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b-\frac{\sqrt{-a b}}{b}}-\frac{a d-b c}{b}}\right)}{}\right.
$$

$$
2 b^{3} \sqrt{-\frac{a d-b c}{b}}
$$



Problem 194: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{x^{3}\left(b x^{2}+a\right) \sqrt{d x^{2}+c}} \mathrm{~d} x
$$

Optimal(type 3, 93 leaves, 7 steps):

$$
\frac{(a d+2 b c) \operatorname{arctanh}\left(\frac{\sqrt{d x^{2}+c}}{\sqrt{c}}\right)}{2 a^{2} c^{3 / 2}}-\frac{b^{3 / 2} \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{d x^{2}+c}}{\sqrt{-a d+b c}}\right)}{a^{2} \sqrt{-a d+b c}}-\frac{\sqrt{d x^{2}+c}}{2 a c x^{2}}
$$

Result(type 3, 384 leaves):

$$
-\frac{\sqrt{d x^{2}+c}}{2 a c x^{2}}+\frac{d \ln \left(\frac{2 c+2 \sqrt{c} \sqrt{d x^{2}+c}}{x}\right)}{2 a c^{3 / 2}}+\frac{b \ln \left(\frac{2 c+2 \sqrt{c} \sqrt{d x^{2}+c}}{x}\right)}{a^{2} \sqrt{c}}
$$

$$
-\frac{b \ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right.}{2 a^{2} \sqrt{-\frac{a d-b c}{b}}}
$$

$$
-\frac{b \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right)}{1}
$$

$$
2 a^{2} \sqrt{-\frac{a d-b c}{b}}
$$

Problem 195: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{2}}{\left(b x^{2}+a\right) \sqrt{d x^{2}+c}} \mathrm{~d} x
$$

Optimal(type 3, 66 leaves, 5 steps):

$$
\frac{\operatorname{arctanh}\left(\frac{x \sqrt{d}}{\sqrt{d x^{2}+c}}\right)}{b \sqrt{d}}-\frac{\arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right) \sqrt{a}}{b \sqrt{-a d+b c}}
$$

Result(type 3, 336 leaves):
$\frac{\ln \left(x \sqrt{d}+\sqrt{d x^{2}+c}\right)}{b \sqrt{d}}$
$b \sqrt{d}$
$\left.+\frac{a \ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right)}{2 \sqrt{-a b} b \sqrt{-\frac{a d-b c}{b}}}\right)$
$-\frac{a \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right)}{2 \sqrt{-a b} b \sqrt{-\frac{a d-b c}{b}}}$

Problem 196: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{x^{2}\left(b x^{2}+a\right) \sqrt{d x^{2}+c}} \mathrm{~d} x
$$

Optimal(type 3, 62 leaves, 4 steps):

$$
-\frac{b \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{a^{3 / 2} \sqrt{-a d+b c}}-\frac{\sqrt{d x^{2}+c}}{a c x}
$$

Result(type 3, 333 leaves):


Problem 197: Result more than twice size of optimal antiderivative.

$$
\int \frac{x}{\left(b x^{2}+a\right)\left(d x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 60 leaves, 4 steps):

$$
-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{d x^{2}+c}}{\sqrt{-a d+b c}}\right) \sqrt{b}}{(-a d+b c)^{3 / 2}}+\frac{1}{(-a d+b c) \sqrt{d x^{2}+c}}
$$

Result(type 3, 617 leaves):
1
$2(a d-b c) \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$

$$
+\frac{\sqrt{-a b} x d}{2 b(a d-b c) c \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}
$$

$$
+\frac{\ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right)}{2(a d-b c) \sqrt{-\frac{a d-b c}{b}}}
$$

$$
2(a d-b c) \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}
$$

$$
-\frac{\sqrt{-a b} x d}{2 b(a d-b c) c \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}
$$

$$
\ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right)
$$

Problem 198: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{x^{4}\left(b x^{2}+a\right)\left(d x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 156 leaves, 6 steps):

$$
\frac{b^{3} \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{a^{5 / 2}(-a d+b c)^{3 / 2}}-\frac{d}{c(-a d+b c) x^{3} \sqrt{d x^{2}+c}}-\frac{(-4 a d+b c) \sqrt{d x^{2}+c}}{3 a c^{2}(-a d+b c) x^{3}}+\frac{(-4 a d+3 b c)(2 a d+b c) \sqrt{d x^{2}+c}}{3 a^{2} c^{3}(-a d+b c) x}
$$

Result(type 3, 761 leaves):
$-\frac{1}{3 a c x^{3} \sqrt{d x^{2}+c}}+\frac{4 d}{3 a c^{2} x \sqrt{d x^{2}+c}}+\frac{8 d^{2} x}{3 a c^{3} \sqrt{d x^{2}+c}}+\frac{b}{a^{2} c x \sqrt{d x^{2}+c}}+\frac{2 b d x}{a^{2} c^{2} \sqrt{d x^{2}+c}}$

$$
\begin{aligned}
& 2 a^{2} \sqrt{-a b}(a d-b c) \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}} \\
& +\frac{b^{2} x d}{2 a^{2}(a d-b c) c \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}} \\
& b^{3} \ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right) \\
& 2 a^{2} \sqrt{-a b}(a d-b c) \sqrt{-\frac{a d-b c}{b}} \\
& b^{3} \\
& 2 a^{2} \sqrt{-a b}(a d-b c) \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}} \\
& b^{2} x d \\
& 2 a^{2}(a d-b c) c \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}} \\
& b^{3} \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right) \\
& 2 a^{2} \sqrt{-a b}(a d-b c) \sqrt{-\frac{a d-b c}{b}}
\end{aligned}
$$

Problem 199: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{2}}{\left(b x^{2}+a\right)\left(d x^{2}+c\right)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 97 leaves, 5 steps):

$$
\frac{x}{3(-a d+b c)\left(d x^{2}+c\right)^{3 / 2}}-\frac{b \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right) \sqrt{a}}{(-a d+b c)^{5 / 2}}+\frac{(a d+2 b c) x}{3 c(-a d+b c)^{2} \sqrt{d x^{2}+c}}
$$

Result(type 3, 1133 leaves):

$$
\begin{aligned}
& \frac{x}{3 b c\left(d x^{2}+c\right)^{3 / 2}}+\frac{2 x}{3 b c^{2} \sqrt{d x^{2}+c}}+\frac{a}{6 \sqrt{-a b}(a d-b c)\left(d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2}} \\
& -\frac{a d x}{6 b(a d-b c) c\left(d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2}}
\end{aligned}
$$

$$
-\frac{a d x}{3 b(a d-b c) c^{2} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}
$$

$a b$
$2 \sqrt{-a b}(a d-b c)^{2} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$

$$
+\frac{a x d}{2(a d-b c)^{2} c \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}
$$

$$
a b \ln \left(\frac{\left.-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b-\frac{\sqrt{-a b}}{b}}}\right)}{x-\frac{a d-b c}{b}}\right)
$$

$$
2 \sqrt{-a b}(a d-b c)^{2} \sqrt{-\frac{a d-b c}{b}}
$$

$$
6 \sqrt{-a b}(a d-b c)\left(d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2}
$$

$6 b(a d-b c) c\left(d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2}$
$a d x$
$3 b(a d-b c) c^{2} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$

$$
+\frac{a b}{2 \sqrt{-a b}(a d-b c)^{2} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}
$$

$$
+\frac{a x d}{2(a d-b c)^{2} c \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}
$$

$$
a b \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right)
$$

$$
2 \sqrt{-a b}(a d-b c)^{2} \sqrt{-\frac{a d-b c}{b}}
$$

Problem 200: Result more than twice size of optimal antiderivative.

$$
\int \frac{x}{\left(b x^{2}+a\right)\left(d x^{2}+c\right)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 82 leaves, 5 steps):

$$
\frac{1}{3(-a d+b c)\left(d x^{2}+c\right)^{3 / 2}}-\frac{b^{3 / 2} \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{d x^{2}+c}}{\sqrt{-a d+b c}}\right)}{(-a d+b c)^{5 / 2}}+\frac{b}{(-a d+b c)^{2} \sqrt{d x^{2}+c}}
$$

Result(type 3, 1085 leaves):

$$
\begin{aligned}
& 6(a d-b c)\left(d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2} \\
& d \sqrt{-a b} x \\
& 6 b(a d-b c) c\left(d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2} \\
& +\frac{d \sqrt{-a b} x}{3 b(a d-b c) c^{2} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}} \\
& +\frac{b}{2(a d-b c)^{2} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}} \\
& -\frac{\sqrt{-a b} x d}{2(a d-b c)^{2} c \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}} \\
& b \ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{x-\frac{\sqrt{-a b}}{b}}}{x\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right) \\
& 2(a d-b c)^{2} \sqrt{-\frac{a d-b c}{b}} \\
& 6(a d-b c)\left(d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2} \\
& d \sqrt{-a b} x \\
& 6 b(a d-b c) c\left(d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2}
\end{aligned}
$$

$$
\begin{aligned}
& d \sqrt{-a b} x \\
& 3 b(a d-b c) c^{2} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}} \\
& +\frac{b}{2(a d-b c)^{2} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}} \\
& +\frac{\sqrt{-a b} x d}{2(a d-b c)^{2} c \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}} \\
& b \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{x+\frac{\sqrt{-a b}}{b}}}{\left.x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right) \\
& 2(a d-b c)^{2} \sqrt{-\frac{a d-b c}{b}}
\end{aligned}
$$

Problem 201: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{x^{4}\left(b x^{2}+a\right)\left(d x^{2}+c\right)^{5 / 2}} d x
$$

Optimal(type 3, 221 leaves, 7 steps):

$$
\begin{aligned}
& -\frac{d}{3 c(-a d+b c) x^{3}\left(d x^{2}+c\right)^{3 / 2}}+\frac{b^{4} \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{a^{5 / 2}(-a d+b c)^{5 / 2}}-\frac{d(-2 a d+3 b c)}{c^{2}(-a d+b c)^{2} x^{3} \sqrt{d x^{2}+c}}-\frac{\left(8 a^{2} d^{2}-12 a c b d+b^{2} c^{2}\right) \sqrt{d x^{2}+c}}{3 a c^{3}(-a d+b c)^{2} x^{3}} \\
& \quad+\frac{(-2 a d+b c)\left(-8 a^{2} d^{2}+8 a c b d+3 b^{2} c^{2}\right) \sqrt{d x^{2}+c}}{3 a^{2} c^{4}(-a d+b c)^{2} x}
\end{aligned}
$$

Result(type 3, 1284 leaves):

$$
-\frac{1}{3 a c x^{3}\left(d x^{2}+c\right)^{3 / 2}}+\frac{2 d}{a c^{2} x\left(d x^{2}+c\right)^{3 / 2}}+\frac{8 d^{2} x}{3 a c^{3}\left(d x^{2}+c\right)^{3 / 2}}+\frac{16 d^{2} x}{3 a c^{4} \sqrt{d x^{2}+c}}+\frac{b}{a^{2} c x\left(d x^{2}+c\right)^{3 / 2}}+\frac{4 b d x}{3 a^{2} c^{2}\left(d x^{2}+c\right)^{3 / 2}}
$$

$$
\begin{aligned}
& +\frac{8 b d x}{3 a^{2} c^{3} \sqrt{d x^{2}+c}}-\frac{b^{3}}{6 a^{2} \sqrt{-a b}(a d-b c)\left(d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2}} \\
& b^{2} d x \\
& 6 a^{2}(a d-b c) c\left(d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2} \\
& b^{2} d x \\
& 3 a^{2}(a d-b c) c^{2} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}} \\
& +\frac{b^{4}}{2 a^{2} \sqrt{-a b}(a d-b c)^{2} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}} \\
& b^{3} x d \\
& 2 a^{2}(a d-b c)^{2} c \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}} \\
& b^{4} \ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{x-\frac{\sqrt{-a b}}{b}}}{x\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right) \\
& 2 a^{2} \sqrt{-a b}(a d-b c)^{2} \sqrt{-\frac{a d-b c}{b}} \\
& +\frac{b^{3}}{6 a^{2} \sqrt{-a b}(a d-b c)\left(d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2}} \\
& b^{2} d x \\
& 6 a^{2}(a d-b c) c\left(d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)^{3 / 2}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{b^{2} d x}{3 a^{2}(a d-b c) c^{2} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}} \\
& b^{4} \\
& 2 a^{2} \sqrt{-a b}(a d-b c)^{2} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}} \\
& -\frac{b^{3} x d}{2 a^{2}(a d-b c)^{2} c \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}} \\
& b^{4} \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right)
\end{aligned}
$$

Problem 202: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{d x^{2}+c}}{x^{2}\left(b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 93 leaves, 5 steps):

$$
-\frac{(-2 a d+3 b c) \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{2 a^{5 / 2} \sqrt{-a d+b c}}-\frac{3 \sqrt{d x^{2}+c}}{2 a^{2} x}+\frac{\sqrt{d x^{2}+c}}{2 a x\left(b x^{2}+a\right)}
$$

Result(type ?, 2617 leaves): Display of huge result suppressed!
Problem 203: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{4}\left(d x^{2}+c\right)^{3 / 2}}{\left(b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 165 leaves, 8 steps):

$$
-\frac{x^{3}\left(d x^{2}+c\right)^{3 / 2}}{2 b\left(b x^{2}+a\right)}+\frac{3\left(8 a^{2} d^{2}-8 a c b d+b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{x \sqrt{d}}{\sqrt{d x^{2}+c}}\right)}{8 b^{4} \sqrt{d}}-\frac{3(-2 a d+b c) \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right) \sqrt{a} \sqrt{-a d+b c}}{2 b^{4}}
$$

$$
+\frac{3(-4 a d+3 b c) x \sqrt{d x^{2}+c}}{8 b^{3}}+\frac{3 d x^{3} \sqrt{d x^{2}+c}}{4 b^{2}}
$$

Result(type ?, 4794 leaves): Display of huge result suppressed!
Problem 204: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{2}\left(d x^{2}+c\right)^{3 / 2}}{\left(b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Optimal (type 3, 123 leaves, 7 steps):

$$
-\frac{x\left(d x^{2}+c\right)^{3 / 2}}{2 b\left(b x^{2}+a\right)}+\frac{(-4 a d+3 b c) \operatorname{arctanh}\left(\frac{x \sqrt{d}}{\sqrt{d x^{2}+c}}\right) \sqrt{d}}{2 b^{3}}+\frac{(-4 a d+b c) \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right) \sqrt{-a d+b c}}{2 b^{3} \sqrt{a}}+\frac{d x \sqrt{d x^{2}+c}}{b^{2}}
$$

Result(type ?, 4684 leaves): Display of huge result suppressed!
Problem 205: Result more than twice size of optimal antiderivative.

$$
\int \frac{x\left(d x^{2}+c\right)^{3 / 2}}{\left(b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 79 leaves, 5 steps):

$$
-\frac{\left(d x^{2}+c\right)^{3 / 2}}{2 b\left(b x^{2}+a\right)}-\frac{3 d \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{d x^{2}+c}}{\sqrt{-a d+b c}}\right) \sqrt{-a d+b c}}{2 b^{5 / 2}}+\frac{3 d \sqrt{d x^{2}+c}}{2 b^{2}}
$$

Result(type ?, 2820 leaves): Display of huge result suppressed!
Problem 206: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(d x^{2}+c\right)^{5 / 2}}{x^{2}\left(b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 142 leaves, 7 steps):

$$
\frac{(-a d+b c)\left(d x^{2}+c\right)^{3 / 2}}{2 a b x\left(b x^{2}+a\right)}-\frac{(-a d+b c)^{3 / 2}(2 a d+3 b c) \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{2 a^{5 / 2} b^{2}}+\frac{d^{5 / 2} \operatorname{arctanh}\left(\frac{x \sqrt{d}}{\sqrt{d x^{2}+c}}\right)}{b^{2}}-\frac{c(-a d+3 b c) \sqrt{d x^{2}+c}}{2 a^{2} x b}
$$

Result(type ?, 7528 leaves): Display of huge result suppressed!
Problem 207: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(d x^{2}+c\right)^{5 / 2}}{x^{3}\left(b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Optimal(type 3, 152 leaves, 8 steps):

$$
\begin{aligned}
& -\frac{c\left(d x^{2}+c\right)^{3 / 2}}{2 a x^{2}\left(b x^{2}+a\right)}+\frac{c^{3 / 2}(-5 a d+4 b c) \operatorname{arctanh}\left(\frac{\sqrt{d x^{2}+c}}{\sqrt{c}}\right)}{2 a^{3}}-\frac{(-a d+b c)^{3 / 2}(a d+4 b c) \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{d x^{2}+c}}{\sqrt{-a d+b c}}\right)}{2 a^{3} b^{3 / 2}} \\
& \quad-\frac{(-a d+b c)(-a d+2 b c) \sqrt{d x^{2}+c}}{2 a^{2} b\left(b x^{2}+a\right)}
\end{aligned}
$$

Result(type ?, 7589 leaves): Display of huge result suppressed!
Problem 208: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{4}}{\left(b x^{2}+a\right)^{2} \sqrt{d x^{2}+c}} \mathrm{~d} x
$$

Optimal(type 3, 110 leaves, 6 steps):

$$
-\frac{(-2 a d+3 b c) \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right) \sqrt{a}}{2 b^{2}(-a d+b c)^{3 / 2}}+\frac{\operatorname{arctanh}\left(\frac{x \sqrt{d}}{\sqrt{d x^{2}+c}}\right)}{b^{2} \sqrt{d}}+\frac{a x \sqrt{d x^{2}+c}}{2 b(-a d+b c)\left(b x^{2}+a\right)}
$$

Result(type 3, 845 leaves):
$\frac{\ln \left(x \sqrt{d}+\sqrt{d x^{2}+c}\right)}{b^{2} \sqrt{d}}-\frac{a \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{4 b^{2}(a d-b c)\left(x-\frac{\sqrt{-a b}}{b}\right)}$


$$
\begin{aligned}
& -\frac{a \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{4 b^{2}(a d-b c)\left(x+\frac{\sqrt{-a b}}{b}\right)} \\
& -\longrightarrow\left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{x+\frac{\sqrt{-a b}}{b}}}{\left.x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right) \\
& 4 b^{3}(a d-b c) \sqrt{-\frac{a d-b c}{b}} \\
& 3 a \ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right) \\
& \left.-\frac{2 a \ln \left(\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}\right.}{x+\frac{\sqrt{-a b}}{b}}\right)
\end{aligned}
$$

Problem 209: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{2}}{\left(b x^{2}+a\right)^{2} \sqrt{d x^{2}+c}} \mathrm{~d} x
$$

Optimal(type 3, 73 leaves, 4 steps):

$$
\frac{c \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{2(-a d+b c)^{3 / 2} \sqrt{a}}-\frac{x \sqrt{d x^{2}+c}}{2(-a d+b c)\left(b x^{2}+a\right)}
$$

## Result(type 3, 816 leaves):

$$
\frac{\sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{4 b(a d-b c)\left(x-\frac{\sqrt{-a b}}{b}\right)}
$$

$$
-\sqrt{-a b} \ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right)
$$

$$
4 b^{2}(a d-b c) \sqrt{-\frac{a d-b c}{b}}
$$

$$
+\frac{\sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{4 b(a d-b c)\left(x+\frac{\sqrt{-a b}}{b}\right)}
$$

$$
+\frac{d \sqrt{-a b} \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right)}{4 b^{2}(a d-b c) \sqrt{-\frac{a d-b c}{b}}}
$$

$$
-\frac{\ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right)}{}
$$

$$
\ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right)
$$

Problem 210: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{x\left(b x^{2}+a\right)^{2} \sqrt{d x^{2}+c}} \mathrm{~d} x
$$

Optimal(type 3, 108 leaves, 7 steps):

$$
\frac{(-3 a d+2 b c) \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{d x^{2}+c}}{\sqrt{-a d+b c}}\right) \sqrt{b}}{2 a^{2}(-a d+b c)^{3 / 2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^{2}+c}}{\sqrt{c}}\right)}{a^{2} \sqrt{c}}+\frac{b \sqrt{d x^{2}+c}}{2 a(-a d+b c)\left(b x^{2}+a\right)}
$$

Result(type 3, 837 leaves):
$-\frac{\ln \left(\frac{2 c+2 \sqrt{c} \sqrt{d x^{2}+c}}{x}\right)}{a^{2} \sqrt{c}}$



$$
\begin{aligned}
& -\frac{b \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{4 a \sqrt{-a b}(a d-b c)\left(x-\frac{\sqrt{-a b}}{b}\right)} \\
& d \ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right) \\
& +\frac{b \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{4 a \sqrt{-a b}(a d-b c)\left(x+\frac{\sqrt{-a b}}{b}\right)} \\
& d \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right)
\end{aligned}
$$

Problem 211: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{3}}{\left(b x^{2}+a\right)^{2}\left(d x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 114 leaves, 5 steps):

$$
-\frac{(a d+2 b c) \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{d x^{2}+c}}{\sqrt{-a d+b c}}\right)}{2(-a d+b c)^{5 / 2} \sqrt{b}}+\frac{a d+2 b c}{2 b(-a d+b c)^{2} \sqrt{d x^{2}+c}}+\frac{a}{2 b(-a d+b c)\left(b x^{2}+a\right) \sqrt{d x^{2}+c}}
$$

Result(type 3, 1455 leaves):

$-\frac{1}{2 b(a d-b c) \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}$
$\sqrt{-a b} x d$
$b^{2}(a d-b c) c \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$

$+\frac{\sqrt{-a b}}{4 b^{2}(a d-b c)\left(x-\frac{\sqrt{-a b}}{b}\right) \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}$
$+\frac{3 a d}{4 b(a d-b c)^{2} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}$
$3 \sqrt{-a b} d^{2} a x$
$4 b^{2}(a d-b c)^{2} c \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$


$$
4 b(a d-b c)^{2} \sqrt{-\frac{a d-b c}{b}}
$$

$\sqrt{-a b}$
$4 b^{2}(a d-b c)\left(x+\frac{\sqrt{-a b}}{b}\right) \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$
$+\frac{3 a d}{4 b(a d-b c)^{2} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}$
$3 \sqrt{-a b} d^{2} a x$
$4 b^{2}(a d-b c)^{2} c \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$
$-3 a d \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right)$
$4 b(a d-b c)^{2} \sqrt{-\frac{a d-b c}{b}}$

Problem 212: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{x\left(b x^{2}+a\right)^{2}\left(d x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 144 leaves, 8 steps):

$$
-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^{2}+c}}{\sqrt{c}}\right)}{a^{2} c^{3 / 2}}+\frac{b^{3 / 2}(-5 a d+2 b c) \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{d x^{2}+c}}{\sqrt{-a d+b c}}\right)}{2 a^{2}(-a d+b c)^{5 / 2}}+\frac{d(2 a d+b c)}{2 a c(-a d+b c)^{2} \sqrt{d x^{2}+c}}+\frac{b}{2 a(-a d+b c)\left(b x^{2}+a\right) \sqrt{d x^{2}+c}}
$$

Result(type 3, 1671 leaves):

$$
\frac{1}{a^{2} c \sqrt{d x^{2}+c}}-\frac{\ln \left(\frac{2 c+2 \sqrt{c} \sqrt{d x^{2}+c}}{x}\right)}{a^{2} c^{3 / 2}}+\frac{b}{2 a^{2}(a d-b c) \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}
$$

$\sqrt{-a b} x d$
$2 a^{2}(a d-b c) c \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$


$$
+\frac{b}{2 a^{2}(a d-b c) \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}
$$

$$
\frac{\sqrt{-a b} x d}{2 a^{2}(a d-b c) c \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}
$$

$$
b \ln \left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right)
$$

b
$4 a \sqrt{-a b}(a d-b c)\left(x-\frac{\sqrt{-a b}}{b}\right) \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$

$$
+\frac{3 d b}{4 a(a d-b c)^{2} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}
$$

$3 d^{2} b x$
$4 \sqrt{-a b}(a d-b c)^{2} c \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$

$$
3 d b \ln \left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right)
$$

$$
4 a(a d-b c)^{2} \sqrt{-\frac{a d-b c}{b}}
$$

$b x d$
$2 a \sqrt{-a b}(a d-b c) c \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$
$b$
$4 a \sqrt{-a b}(a d-b c)\left(x+\frac{\sqrt{-a b}}{b}\right) \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$


Problem 213: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{x^{4}\left(b x^{2}+a\right)^{2}\left(d x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 249 leaves, 7 steps):

$$
\begin{aligned}
& \frac{b^{3}(-8 a d+5 b c) \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{2 a^{7 / 2}(-a d+b c)^{5 / 2}}+\frac{d(2 a d+b c)}{2 a c(-a d+b c)^{2} x^{3} \sqrt{d x^{2}+c}}+\frac{2 a(-a d+b c) x^{3}\left(b x^{2}+a\right) \sqrt{d x^{2}+c}}{2} \\
& -\frac{\left(8 a^{2} d^{2}-4 a c b d+5 b^{2} c^{2}\right) \sqrt{d x^{2}+c}}{6 a^{2} c^{2}(-a d+b c)^{2} x^{3}}+\frac{\left(16 a^{3} d^{3}-8 a^{2} b c d^{2}-14 a b^{2} c^{2} d+15 b^{3} c^{3}\right) \sqrt{d x^{2}+c}}{6 a^{3} c^{3}(-a d+b c)^{2} x}
\end{aligned}
$$

Result(type 3, 1607 leaves):
$-\frac{1}{3 a^{2} c x^{3} \sqrt{d x^{2}+c}}+\frac{4 d}{3 a^{2} c^{2} x \sqrt{d x^{2}+c}}+\frac{8 d^{2} x}{3 a^{2} c^{3} \sqrt{d x^{2}+c}}+\frac{2 b}{a^{3} c x \sqrt{d x^{2}+c}}+\frac{4 b d x}{a^{3} c^{2} \sqrt{d x^{2}+c}}$
$4 a^{3}(a d-b c)\left(x-\frac{\sqrt{-a b}}{b}\right) \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$
$3 b^{2} d \sqrt{-a b}$
$4 a^{3}(a d-b c)^{2} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$
$3 b^{2} d^{2} x$
$4 a^{2}(a d-b c)^{2} c \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$
$-\frac{1}{4 a^{3}(a d-b c)^{2} \sqrt{-\frac{a d-b c}{b}}}\left(3 b^{2} d \sqrt{-a b} \ln \left(\frac{1}{x-\frac{\sqrt{-a b}}{b}}\left(-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}\right.\right.\right.$
$\left.+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{\left.d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)}\right)$
$3 b^{2} x d$
$4 a^{3}(a d-b c) c \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$
$b^{2}$
$4 a^{3}(a d-b c)\left(x+\frac{\sqrt{-a b}}{b}\right) \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$
$3 b^{2} d \sqrt{-a b}$
$4 a^{3}(a d-b c)^{2} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}$
$3 b^{2} d^{2} x$

$$
\begin{aligned}
& 4 a^{2}(a d-b c)^{2} c \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}} \\
& +\frac{1}{4 a^{3}(a d-b c)^{2} \sqrt{-\frac{a d-b c}{b}}}\left(3 b ^ { 2 } d \sqrt { - a b } \operatorname { l n } \left(\frac { 1 } { x + \frac { \sqrt { - a b } } { b } } \left(-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}\right.\right.\right. \\
& \left.+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{\left.d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}\right)}\right) \\
& +\frac{3 b^{2} x d}{4 a^{3}(a d-b c) c \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}} \\
& 5 b^{3} \\
& 4 a^{3} \sqrt{-a b}(a d-b c) \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}} \\
& 5 b^{3} \ln \left(\frac{\left.-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{x-\frac{\sqrt{-a b}}{b}}-\frac{a d-b c}{b}}\right)}{x}\right) \\
& 4 a^{3} \sqrt{-a b}(a d-b c) \sqrt{-\frac{a d-b c}{b}} \\
& +\frac{5 b^{3}}{4 a^{3} \sqrt{-a b}(a d-b c) \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}
\end{aligned}
$$



$$
4 a^{3} \sqrt{-a b}(a d-b c) \sqrt{-\frac{a d-b c}{b}}
$$

Problem 214: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{2}}{\left(b x^{2}+a\right)^{2}\left(d x^{2}+c\right)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 139 leaves, 6 steps):

$$
-\frac{5 d x}{6(-a d+b c)^{2}\left(d x^{2}+c\right)^{3 / 2}}-\frac{x}{2(-a d+b c)\left(b x^{2}+a\right)\left(d x^{2}+c\right)^{3 / 2}}+\frac{b(4 a d+b c) \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{2(-a d+b c)^{7 / 2} \sqrt{a}}-\frac{d(2 a d+13 b c) x}{6 c(-a d+b c)^{3} \sqrt{d x^{2}+c}}
$$

Result(type ?, 2368 leaves): Display of huge result suppressed!
Problem 215: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{x^{2}\left(b x^{2}+a\right)^{2}\left(d x^{2}+c\right)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 251 leaves, 7 steps):

$$
\begin{aligned}
& \frac{d(2 a d+3 b c)}{6 a c(-a d+b c)^{2} x\left(d x^{2}+c\right)^{3 / 2}}+\frac{b}{2 a(-a d+b c) x\left(b x^{2}+a\right)\left(d x^{2}+c\right)^{3 / 2}}-\frac{b^{3}(-8 a d+3 b c) \arctan \left(\frac{x \sqrt{-a d+b c}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{2 a^{5 / 2}(-a d+b c)^{7 / 2}} \\
& \quad+\frac{d\left(-8 a^{2} d^{2}+20 a c b d+3 b^{2} c^{2}\right)}{6 a c^{2}(-a d+b c)^{3} x \sqrt{d x^{2}+c}}-\frac{\left(-16 a^{3} d^{3}+40 a^{2} b c d^{2}-18 a b^{2} c^{2} d+9 b^{3} c^{3}\right) \sqrt{d x^{2}+c}}{6 a^{2} c^{3}(-a d+b c)^{3} x}
\end{aligned}
$$

Result(type ?, 2512 leaves): Display of huge result suppressed!
Problem 221: Result more than twice size of optimal antiderivative.

$$
\int \frac{(e x)^{3 / 2}\left(B x^{2}+A\right)}{\left(b x^{2}+a\right)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 192 leaves, 4 steps):
$\frac{(A b-a B)(e x)^{5 / 2}}{3 a b e\left(b x^{2}+a\right)^{3 / 2}}-\frac{(A b+5 a B) e \sqrt{e x}}{6 a b^{2} \sqrt{b x^{2}+a}}$

$$
+\frac{(A b+5 a B) e^{3 / 2} \sqrt{\cos \left(2 \arctan \left(\frac{b^{1 / 4} \sqrt{e x}}{a^{1 / 4} \sqrt{e}}\right)\right)^{2}} \operatorname{EllipticF}\left(\sin \left(2 \arctan \left(\frac{b^{1 / 4} \sqrt{e x}}{a^{1 / 4} \sqrt{e}}\right)\right), \frac{\sqrt{2}}{2}\right)(\sqrt{a}+x \sqrt{b}) \sqrt{\frac{b x^{2}+a}{(\sqrt{a}+x \sqrt{b})^{2}}}}{12 \cos \left(2 \arctan \left(\frac{b^{1 / 4} \sqrt{e x}}{a^{1 / 4} \sqrt{e}}\right)\right) a^{5 / 4} b^{9 / 4} \sqrt{b x^{2}+a}}
$$

Result(type 4, 428 leaves):

$$
\begin{aligned}
& \frac{1}{12 x a b^{3}\left(b x^{2}+a\right)^{3 / 2}}\left(\left(A \sqrt{2} \sqrt{-a b} \sqrt{\frac{b x+\sqrt{-a b}}{\sqrt{-a b}}} \sqrt{\frac{-b x+\sqrt{-a b}}{\sqrt{-a b}}} \sqrt{-\frac{x b}{\sqrt{-a b}}} \text { EllipticF }\left(\sqrt{\frac{b x+\sqrt{-a b}}{\sqrt{-a b}}}, \frac{\sqrt{2}}{2}\right) x^{2} b^{2}\right.\right. \\
& \quad+5 B \sqrt{2} \sqrt{-a b} \sqrt{\frac{b x+\sqrt{-a b}}{\sqrt{-a b}}} \sqrt{\frac{-b x+\sqrt{-a b}}{\sqrt{-a b}}} \sqrt{\frac{-\frac{x b}{\sqrt{-a b}}}{}} \operatorname{EllipticF}\left(\sqrt{\left.\frac{b x+\sqrt{-a b}}{\sqrt{-a b}}, \frac{\sqrt{2}}{2}\right) x^{2} a b}\right. \\
& \quad+A \sqrt{2} \sqrt{-a b} \sqrt{\frac{b x+\sqrt{-a b}}{\sqrt{-a b}}} \sqrt{\frac{-b x+\sqrt{-a b}}{\sqrt{-a b}}} \sqrt{-\frac{x b}{\sqrt{-a b}}} \operatorname{EllipticF}\left(\sqrt{\frac{b x+\sqrt{-a b}}{\sqrt{-a b}}, \frac{\sqrt{2}}{2}}\right) a b \\
& \quad+5 B \sqrt{2} \sqrt{-a b} \sqrt{\frac{b x+\sqrt{-a b}}{\sqrt{-a b}}} \sqrt{\frac{-b x+\sqrt{-a b}}{\sqrt{-a b}}} \sqrt{-\frac{x b}{\sqrt{-a b}}} \operatorname{EllipticF}\left(\sqrt{\frac{b x+\sqrt{-a b}}{\sqrt{-a b}}}, \frac{\sqrt{2}}{2}\right) a^{2}+2 A x^{3} b^{3}-14 B x^{3} a b^{2}-2 A x a b^{2} \\
& \left.\left.\quad-10 B x a^{2} b\right) e \sqrt{e x}\right)
\end{aligned}
$$

Problem 231: Result more than twice size of optimal antiderivative.

$$
\int \frac{(e x)^{5} / 2 \sqrt{-d x^{2}+c}}{-b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 4, 308 leaves, 15 steps):

$$
\begin{aligned}
& -\frac{2 e(e x)^{3 / 2} \sqrt{-d x^{2}+c}}{5 b}-\frac{2 c^{3 / 4}(-5 a d+2 b c) e^{5 / 2} \operatorname{EllipticE}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{5 b^{2} d^{3} / 4 \sqrt{-d x^{2}+c}} \\
& +\frac{2 c^{3 / 4}(-5 a d+2 b c) e^{5 / 2} \operatorname{EllipticF}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}} c^{1 / 4}(-a d+b c) e^{5 / 2} \operatorname{EllipticPi}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}},-\frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{a} \sqrt{1-\frac{d x^{2}}{c}}}{5 b^{2} d^{3 / 4} \sqrt{-d x^{2}+c}} \\
& +\frac{c^{1 / 4}(-a d+b c) e^{5 / 2} \operatorname{EllipticPi}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{a} \sqrt{1-\frac{d x^{2}}{c}}}{b^{5 / 2} d^{1 / 4} \sqrt{-d x^{2}+c}}
\end{aligned}
$$

Result(type 4, 1490 leaves):
$\frac{1}{10 x b^{2}\left(d x^{2}-c\right)(\sqrt{a b} d+\sqrt{c d} b)(\sqrt{c d} b-\sqrt{a b} d)}\left(e^{2} \sqrt{e x} \sqrt{-d x^{2}+c}\left(5 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b}\right.\right.\right.$, $\left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a^{2} b c d^{2}-5 \sqrt{a b} \sqrt{c d} \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b}\right.$,
$\left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a^{2} d^{2}-5 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b}\right.$,
$\left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b^{2} c^{2} d+5 \sqrt{a b} \sqrt{c d} \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b}\right.$,
$\left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b c d+5 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d}\right.$,
$\left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a^{2} b c d^{2}+5 \sqrt{a b} \sqrt{c d} \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d}\right.$,
$\left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a^{2} d^{2}-5 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d}\right.$,
$\left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b^{2} c^{2} d-5 \sqrt{a b} \sqrt{c d} \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d}\right.$,
$\left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b c d-20 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \operatorname{EllipticE}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a^{2} b c d^{2}$
$+28 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}}$ EllipticE $\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b^{2} c^{2} d-8 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}}$ EllipticE $\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}\right.$,
$\left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} b^{3} c^{3}+10 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \operatorname{EllipticF}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a^{2} b c d^{2}$
$-14 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}}$ EllipticF $\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b^{2} c^{2} d+4 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}}$ EllipticF $\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}\right.$,
$\left.\left.\left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} b^{3} c^{3}+4 x^{4} a b^{2} d^{3}-4 x^{4} b^{3} c d^{2}-4 x^{2} a b^{2} c d^{2}+4 x^{2} b^{3} c^{2} d\right)\right)$

Problem 232: Result more than twice size of optimal antiderivative.

$$
\int \frac{(e x)^{3 / 2} \sqrt{-d x^{2}+c}}{-b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 4, 237 leaves, 10 steps):

$$
\begin{gathered}
-\frac{2 e \sqrt{e x} \sqrt{-d x^{2}+c}}{3 b}-\frac{2 c^{1 / 4}(-3 a d+2 b c) e^{3 / 2} \operatorname{EllipticF}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{3 b^{2} d^{1 / 4} \sqrt{-d x^{2}+c}} \\
+\frac{c^{1 / 4}(-a d+b c) e^{3 / 2} \operatorname{EllipticPi}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}},-\frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{b^{2} d^{1 / 4} \sqrt{-d x^{2}+c}} \\
+\frac{c^{1 / 4}(-a d+b c) e^{3 / 2} \operatorname{EllipticPi}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{b^{2} d^{1 / 4} \sqrt{-d x^{2}+c}}
\end{gathered}
$$

Result(type 4, 1285 leaves):

$$
\begin{aligned}
& \frac{1}{6 b x\left(d x^{2}-c\right) \sqrt{a b}(\sqrt{a b} d+\sqrt{c d} b)(\sqrt{c d} b-\sqrt{a b} d)}\left(e \sqrt { e x } \sqrt { - d x ^ { 2 } + c } \left(6 \sqrt { 2 } \text { EllipticF } \left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}},\right.\right.\right. \\
& \left.\frac{\sqrt{2}}{2}\right) a^{2} d^{2} \sqrt{a b} \sqrt{c d} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}}-10 \sqrt{2} \text { EllipticF }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) a b c d \sqrt{a b} \sqrt{c d} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}}+4 \sqrt{2} \text { EllipticF }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) b^{2} c^{2} \sqrt{a b} \sqrt{c d} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}}+3 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b}\right. \text {, } \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a^{2} b c d^{2}-3 \sqrt{a b} \sqrt{c d} \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a^{2} d^{2}-3 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b^{2} c^{2} d+3 \sqrt{a b} \sqrt{c d} \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b c d-3 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a^{2} b c d^{2}-3 \sqrt{a b} \sqrt{c d} \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a^{2} d^{2}+3 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b^{2} c^{2} d+3 \sqrt{a b} \sqrt{c d} \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right. \\
& \left.\left.\left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b c d+4 x^{3} a b d^{3} \sqrt{a b}-4 x^{3} b^{2} c d^{2} \sqrt{a b}-4 x a b c d^{2} \sqrt{a b}+4 x b^{2} c^{2} d \sqrt{a b}\right)\right)
\end{aligned}
$$

Problem 233: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(-d x^{2}+c\right)^{3 / 2}}{\left(-b x^{2}+a\right) \sqrt{e x}} \mathrm{~d} x
$$

Optimal(type 4, 250 leaves, 10 steps):

$$
\begin{aligned}
& \frac{2 d \sqrt{e x} \sqrt{-d x^{2}+c}}{3 b e}+\frac{2 c^{1 / 4} d^{3 / 4}(-3 a d+5 b c) \text { EllipticF }\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{3 b^{2} \sqrt{e} \sqrt{-d x^{2}+c}} \\
& \quad+\frac{c^{1 / 4}(-a d+b c)^{2} \operatorname{EllipticPi}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}},-\frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{a b^{2} d^{1 / 4} \sqrt{e} \sqrt{-d x^{2}+c}}+\frac{c^{1 / 4}(-a d+b c)^{2} \operatorname{EllipticPi}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{a b^{2} d^{1 / 4} \sqrt{e} \sqrt{-d x^{2}+c}}
\end{aligned}
$$

Result(type 4, 1720 leaves):

$$
\begin{aligned}
& \frac{1}{6 b \sqrt{e x}\left(d x^{2}-c\right) \sqrt{a b}(\sqrt{a b} d+\sqrt{c d} b)(\sqrt{c d} b-\sqrt{a b} d)}\left(\sqrt { - d x ^ { 2 } + c } d \left(3 \sqrt { 2 } \sqrt { - \frac { x d } { \sqrt { c d } } } \operatorname { E l l i p t i c P i } \left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right.\right.\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a^{2} b c d^{2}+3 \sqrt{a b} \sqrt{c d} \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a^{2} d^{2}-6 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b^{2} c^{2} d-6 \sqrt{a b} \sqrt{c d} \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b c d+3 \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{2} b^{3} c^{3} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}}+3 \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{2} b^{2} c^{2} \sqrt{c d} \sqrt{a b} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}}-6 \sqrt{2} \text { EllipticF }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) a^{2} d^{2} \sqrt{a b} \sqrt{c d} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}}+16 \sqrt{2} \text { EllipticF }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) a b c d \sqrt{a b} \sqrt{c d} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}}-10 \sqrt{2} \text { EllipticF }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}\right. \text {, } \\
& \left.\frac{\sqrt{2}}{2}\right) b^{2} c^{2} \sqrt{a b} \sqrt{c d} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}}-3 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a^{2} b c d^{2}+3 \sqrt{a b} \sqrt{c d} \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a^{2} d^{2}+6 \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b^{2} c^{2} d-6 \sqrt{a b} \sqrt{c d} \sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b c d-3 \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{2} b^{3} c^{3} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}}+3 \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\left.\left.\frac{\sqrt{2}}{2}\right) \sqrt{2} b^{2} c^{2} \sqrt{c d} \sqrt{a b} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}}-4 x^{3} a b d^{3} \sqrt{a b}+4 x^{3} b^{2} c d^{2} \sqrt{a b}+4 x a b c d^{2} \sqrt{a b}-4 x b^{2} c^{2} d \sqrt{a b}\right)\right)
\end{aligned}
$$

Problem 234: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(-d x^{2}+c\right)^{3 / 2}}{(e x)^{5 / 2}\left(-b x^{2}+a\right)} d x
$$

Optimal(type 4, 252 leaves, 10 steps):

$$
-\frac{2 c \sqrt{-d x^{2}+c}}{3 a e(e x)^{3 / 2}}+\frac{2 c^{1 / 4} d^{3 / 4}(-3 a d+b c) \text { EllipticF }\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{3 a b e^{5 / 2} \sqrt{-d x^{2}+c}}
$$

$$
+\frac{c^{1 / 4}(-a d+b c)^{2} \operatorname{EllipticPi}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}},-\frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{a^{2} b d^{1 / 4} e^{5 / 2} \sqrt{-d x^{2}+c}}+\frac{c^{1 / 4}(-a d+b c)^{2} \operatorname{EllipticPi}\left(\frac{d^{1} / 4}{c^{1 / 4} \sqrt{e x}}, \frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{a^{2} b d^{1 / 4} e^{5 / 2} \sqrt{-d x^{2}+c}}
$$

Result(type 4, 1739 leaves):

$$
\begin{aligned}
& \frac{1}{6 x a e^{2} \sqrt{e x}\left(d x^{2}-c\right) \sqrt{a b}(\sqrt{a b} d+\sqrt{c d} b)(\sqrt{c d} b-\sqrt{a b} d)}\left(\sqrt { - d x ^ { 2 } + c } d \left(3 \sqrt { 2 } \text { EllipticPi } \left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right.\right.\right. \\
& \left.\frac{\sqrt{2}}{2}\right) x a^{2} b c d^{2} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}}+3 \sqrt{2} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) x a^{2} d^{2} \sqrt{a b} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{c d}-6 \sqrt{2} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) x a b^{2} c^{2} d \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}}-6 \sqrt{2} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) x a b c d \sqrt{a b} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{c d}+3 \sqrt{2} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) x b^{3} c^{3} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}}+3 \sqrt{2} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) x b^{2} c^{2} \sqrt{a b} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{c d}-6 \sqrt{2} \text { EllipticF }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) x a^{2} d^{2} \sqrt{a b} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{c d}+8 \sqrt{2} \text { EllipticF }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}\right. \text {, } \\
& \left.\frac{\sqrt{2}}{2}\right) x a b c d \sqrt{a b} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{c d}-2 \sqrt{2} \text { EllipticF }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}},\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left.\frac{\sqrt{2}}{2}\right) x b^{2} c^{2} \sqrt{a b} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{c d}-3 \sqrt{2} \operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) x a^{2} b c d^{2} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}}+3 \sqrt{2} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) x a^{2} d^{2} \sqrt{a b} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{c d}+6 \sqrt{2} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) x a b^{2} c^{2} d \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}}-6 \sqrt{2} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) x a b c d \sqrt{a b} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{c d}-3 \sqrt{2} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) x b^{3} c^{3} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}}+3 \sqrt{2} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\left.\left.\frac{\sqrt{2}}{2}\right) x b^{2} c^{2} \sqrt{a b} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{c d}+4 x^{2} a b c d^{2} \sqrt{a b}-4 x^{2} b^{2} c^{2} d \sqrt{a b}-4 a b c^{2} d \sqrt{a b}+4 b^{2} c^{3} \sqrt{a b}\right)\right)
\end{aligned}
$$

Problem 235: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(-d x^{2}+c\right)^{3 / 2}}{(e x)^{7 / 2}\left(-b x^{2}+a\right)} d x
$$

Optimal(type 4, 347 leaves, 16 steps):

$$
\begin{aligned}
& -\frac{2 c \sqrt{-d x^{2}+c}}{5 a e(e x)^{5 / 2}}-\frac{2(-7 a d+5 b c) \sqrt{-d x^{2}+c}}{5 a^{2} e^{3} \sqrt{e x}}-\frac{2 c^{3 / 4} d^{1 / 4}(-7 a d+5 b c) \operatorname{EllipticE}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{} \begin{array}{l}
+\frac{2 c^{3 / 4} d^{1 / 4}(-7 a d+5 b c) \operatorname{EllipticF}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{5 a^{2} e^{7 / 2 \sqrt{-d x^{2}+c}}}-\frac{5 a^{2} e^{7 / 2} \sqrt{-d x^{2}+c}}{c^{1 / 4}(-a d+b c)^{2} \operatorname{EllipticPi}\left(\frac{d^{1} / 4 \sqrt{e x}}{c^{1 / 4} \sqrt{e}},-\frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}} \\
\quad+\frac{c^{1 / 4}(-a d+b c)^{2} \operatorname{EllipticPi}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{a^{5 / 2 d^{1 / 4} e^{7 / 2} \sqrt{b} \sqrt{-d x^{2}+c}}} \\
a^{5 / 2} d^{1 / 4} e^{7 / 2 \sqrt{b} \sqrt{-d x^{2}+c}}
\end{array}
\end{aligned}
$$

Result(type ?, 2027 leaves): Display of huge result suppressed!

Problem 237: Result more than twice size of optimal antiderivative.

$$
\int \frac{(e x)^{3 / 2}}{\left(-b x^{2}+a\right)\left(-d x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 240 leaves, 10 steps):
$-\frac{e \sqrt{e x}}{(-a d+b c) \sqrt{-d x^{2}+c}}-\frac{c^{1 / 4} e^{3 / 2} \operatorname{EllipticF}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{d^{1 / 4}(-a d+b c) \sqrt{-d x^{2}+c}}+\frac{c^{1 / 4} e^{3 / 2} \operatorname{EllipticPi}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}},-\frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{d^{1 / 4}(-a d+b c) \sqrt{-d x^{2}+c}}$

$$
+\frac{c^{1 / 4} e^{3 / 2} \text { EllipticPi }\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{d^{1 / 4}(-a d+b c) \sqrt{-d x^{2}+c}}
$$

Result(type 4, 701 leaves):

$$
\begin{aligned}
& -\frac{1}{2 x(\sqrt{c d} b-\sqrt{a b} d)(\sqrt{a b} d+\sqrt{c d} b) \sqrt{a b}(a d-b c)\left(d x^{2}-c\right)}\left(b \left(\sqrt { 2 } \sqrt { - \frac { x d } { \sqrt { c d } } } \text { EllipticPi } \left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right.\right.\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b c d+\operatorname{EllipticPi}\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{c d} b-\sqrt{a b} d},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{2} a d \sqrt{a b} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{c d}-\text { EllipticF }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}\right. \text {, } \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{2} a d \sqrt{a b} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{c d}+\text { EllipticF }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}\right. \text {, } \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{2} b c \sqrt{a b} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{c d}-\sqrt{2} \sqrt{-\frac{x d}{\sqrt{c d}}} \text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\frac{\sqrt{2}}{2}\right) \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} a b c d+\text { EllipticPi }\left(\sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}}, \frac{\sqrt{c d} b}{\sqrt{a b} d+\sqrt{c d} b},\right. \\
& \left.\left.\left.\frac{\sqrt{2}}{2}\right) \sqrt{2} a d \sqrt{a b} \sqrt{\frac{d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{\frac{-d x+\sqrt{c d}}{\sqrt{c d}}} \sqrt{-\frac{x d}{\sqrt{c d}}} \sqrt{c d}-2 x a d^{2} \sqrt{a b}+2 x b c d \sqrt{a b}\right) \sqrt{-d x^{2}+c} e \sqrt{e x}\right)
\end{aligned}
$$

Problem 238: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{-d x^{2}+c}}{\sqrt{e x}\left(-b x^{2}+a\right)^{2}} \mathrm{~d} x
$$

Optimal(type 4, 253 leaves, 10 steps):
$\frac{\sqrt{e x} \sqrt{-d x^{2}+c}}{2 a e\left(-b x^{2}+a\right)}+\frac{c^{1 / 4} d^{3} / 4 \operatorname{EllipticF}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{2 a b \sqrt{e} \sqrt{-d x^{2}+c}}+\frac{c^{1 / 4}(-a d+3 b c) \operatorname{EllipticPi}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}},-\frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{4 a^{2} b d^{1 / 4} \sqrt{e} \sqrt{-d x^{2}+c}}$

$$
+\frac{c^{1 / 4}(-a d+3 b c) \text { EllipticPi }\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{4 a^{2} b d^{1 / 4} \sqrt{e} \sqrt{-d x^{2}+c}}
$$

Result(type ?, 2250 leaves): Display of huge result suppressed!
Problem 239: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(-d x^{2}+c\right)^{3 / 2}}{(e x)^{3 / 2}\left(-b x^{2}+a\right)^{2}} d x
$$

Optimal(type 4, 403 leaves, 16 steps):
$-\frac{(-a d+5 b c) \sqrt{-d x^{2}+c}}{2 a^{2} b e \sqrt{e x}}+\frac{(-a d+b c) \sqrt{-d x^{2}+c}}{2 a b e\left(-b x^{2}+a\right) \sqrt{e x}}-\frac{c^{3 / 4} d^{1 / 4}(-a d+5 b c) \text { EllipticE }\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{2 a^{2} b e^{3 / 2} \sqrt{-d x^{2}+c}}$
$+\frac{c^{3 / 4} d^{1 / 4}(-a d+5 b c) \text { EllipticF }\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{2 a^{2} b e^{3 / 2} \sqrt{-d x^{2}+c}}$

$$
-\left(\frac{c^{1 / 4}\left(-a^{2} d^{2}-4 a c b d+5 b^{2} c^{2}\right) \text { EllipticPi }\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}},-\frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{-}\right.
$$

$$
4 a^{5 / 2} b^{3 / 2} d^{1 / 4} e^{3 / 2} \sqrt{-d x^{2}+c}
$$

$$
+\frac{c^{1 / 4}\left(-a^{2} d^{2}-4 a c b d+5 b^{2} c^{2}\right) \text { EllipticPi }\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{4 a^{5 / 2} b^{3 / 2} d^{1 / 4} e^{3 / 2} \sqrt{-d x^{2}+c}}
$$

Result(type ?, 3878 leaves): Display of huge result suppressed!
Problem 240: Result more than twice size of optimal antiderivative.

$$
\int \frac{(e x)^{7 / 2}}{\left(-b x^{2}+a\right)^{2}\left(-d x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 332 leaves, 11 steps):
$\frac{(a d+2 b c) e^{3} \sqrt{e x}}{2 b(-a d+b c)^{2} \sqrt{-d x^{2}+c}}+\frac{a e^{3} \sqrt{e x}}{2 b(-a d+b c)\left(-b x^{2}+a\right) \sqrt{-d x^{2}+c}}+\frac{c^{1 / 4}(a d+2 b c) e^{7 / 2} \operatorname{EllipticF}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{2 b d^{1 / 4}(-a d+b c)^{2} \sqrt{-d x^{2}+c}}$
$-\frac{c^{1 / 4}(a d+5 b c) e^{7 / 2} \text { EllipticPi }\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}},-\frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{}$
$4 b d^{1 / 4}(-a d+b c)^{2} \sqrt{-d x^{2}+c}$
$-\frac{c^{1 / 4}(a d+5 b c) e^{7 / 2} \operatorname{EllipticPi}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{}$

$$
4 b d^{1 / 4}(-a d+b c)^{2} \sqrt{-d x^{2}+c}
$$

Result(type ?, 2529 leaves): Display of huge result suppressed!
Problem 241: Result more than twice size of optimal antiderivative.

$$
\int \frac{(e x)^{3 / 2}}{\left(-b x^{2}+a\right)^{2}\left(-d x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 303 leaves, 11 steps):
$\frac{3 d e \sqrt{e x}}{2(-a d+b c)^{2} \sqrt{-d x^{2}+c}}+\frac{e \sqrt{e x}}{2(-a d+b c)\left(-b x^{2}+a\right) \sqrt{-d x^{2}+c}}+\frac{3 c^{1 / 4} d^{3 / 4} e^{3 / 2} \operatorname{EllipticF}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{2(-a d+b c)^{2} \sqrt{-d x^{2}+c}}$
$-\frac{c^{1 / 4}(5 a d+b c) e^{3 / 2} \operatorname{EllipticPi}\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}},-\frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{}$

$$
4 a d^{1 / 4}(-a d+b c)^{2} \sqrt{-d x^{2}+c}
$$

$$
-c^{c^{1 / 4}(5 a d+b c) e^{3 / 2} \text { EllipticPi }\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}
$$

$$
4 a d^{1 / 4}(-a d+b c)^{2} \sqrt{-d x^{2}+c}
$$

Result(type ?, 2276 leaves): Display of huge result suppressed!
Problem 242: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{(e x)^{3 / 2}\left(-b x^{2}+a\right)^{2}\left(-d x^{2}+c\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 506 leaves, 17 steps):
$\frac{d(2 a d+b c)}{2 a c(-a d+b c)^{2} e \sqrt{e x} \sqrt{-d x^{2}+c}}+\frac{b}{2 a(-a d+b c) e\left(-b x^{2}+a\right) \sqrt{e x} \sqrt{-d x^{2}+c}}-\frac{\left(6 a^{2} d^{2}-8 a c b d+5 b^{2} c^{2}\right) \sqrt{-d x^{2}+c}}{2 a^{2} c^{2}(-a d+b c)^{2} e \sqrt{e x}}$

$$
-\frac{d^{1 / 4}\left(6 a^{2} d^{2}-8 a c b d+5 b^{2} c^{2}\right) \text { EllipticE }\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{2 a^{2} c^{5 / 4}(-a d+b c)^{2} e^{3 / 2} \sqrt{-d x^{2}+c}}+\frac{d^{1 / 4}\left(6 a^{2} d^{2}-8 a c b d+5 b^{2} c^{2}\right) \text { EllipticF }\left(\frac{d^{1} / 4 \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{2 a^{2} c^{5 / 4}(-a d+b c)^{2} e^{3 / 2} \sqrt{-d x^{2}+c}}
$$

$$
-\underline{b^{3 / 2} c^{1 / 4}(-11 a d+5 b c) \text { EllipticPi }\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}},-\frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}
$$

$$
4 a^{5 / 2} d^{1 / 4}(-a d+b c)^{2} e^{3 / 2} \sqrt{-d x^{2}+c}
$$

$$
+\frac{b^{3 / 2} c^{1 / 4}(-11 a d+5 b c) \text { EllipticPi }\left(\frac{d^{1 / 4} \sqrt{e x}}{c^{1 / 4} \sqrt{e}}, \frac{\sqrt{b} \sqrt{c}}{\sqrt{a} \sqrt{d}}, \mathrm{I}\right) \sqrt{1-\frac{d x^{2}}{c}}}{4 a^{5 / 2} d^{1 / 4}(-a d+b c)^{2} e^{3 / 2} \sqrt{-d x^{2}+c}}
$$

Result(type ?, 3384 leaves): Display of huge result suppressed!
Problem 243: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{b x^{2}+a}}{x \sqrt{d x^{2}+c}} d x
$$

Optimal(type 3, 68 leaves, 8 steps):

$$
-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{b x^{2}+a}}{\sqrt{a} \sqrt{d x^{2}+c}}\right) \sqrt{a}}{\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{b x^{2}+a}}{\sqrt{b} \sqrt{d x^{2}+c}}\right) \sqrt{b}}{\sqrt{d}}
$$

Result(type 3, 176 leaves):

$$
\begin{aligned}
& -\frac{1}{2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d} \sqrt{a c}}\left(\sqrt { b x ^ { 2 } + a } \sqrt { d x ^ { 2 } + c } \left(a \ln \left(\frac{x^{2} a d+b c x^{2}+2 \sqrt{a c} \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c}+2 a c}{x^{2}}\right) \sqrt{b d}\right.\right. \\
& \left.\left.-\ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) b \sqrt{a c}\right)\right)
\end{aligned}
$$

Problem 245: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{3}\left(b x^{2}+a\right)^{3 / 2}}{\sqrt{d x^{2}+c}} \mathrm{~d} x
$$

Optimal(type 3, 155 leaves, 7 steps):

$$
-\frac{(-a d+b c)^{2}(a d+5 b c) \operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{b x^{2}+a}}{\sqrt{b} \sqrt{d x^{2}+c}}\right)}{16 b^{3 / 2} d^{7 / 2}}-\frac{(a d+5 b c)\left(b x^{2}+a\right)^{3 / 2} \sqrt{d x^{2}+c}}{24 b d^{2}}+\frac{\left(b x^{2}+a\right)^{5 / 2} \sqrt{d x^{2}+c}}{6 b d}
$$

$$
+\frac{(-a d+b c)(a d+5 b c) \sqrt{b x^{2}+a} \sqrt{d x^{2}+c}}{16 b d^{3}}
$$

Result(type 3, 531 leaves):
$-\frac{1}{96 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} d^{3} b \sqrt{b d}}\left(\sqrt{b x^{2}+a} \sqrt{d x^{2}+c}\left(-16 x^{4} b^{2} d^{2} \sqrt{b d} \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c}\right.\right.$
$-28 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} x^{2} a d^{2} b \sqrt{b d}+20 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} x^{2} b^{2} c d \sqrt{b d}$
$+3 \ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) a^{3} d^{3}+9 \ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) a^{2} c d^{2} b$
$-27 \ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) a c^{2} b^{2} d+15 b^{3} \ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) c^{3}$
$\left.\left.-6 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} a^{2} d^{2} \sqrt{b d}+44 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} a c d b \sqrt{b d}-30 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} c^{2} b^{2} \sqrt{b d}\right)\right)$

Problem 246: Result more than twice size of optimal antiderivative.

$$
\int \frac{x\left(b x^{2}+a\right)^{3 / 2}}{\sqrt{d x^{2}+c}} \mathrm{~d} x
$$

Optimal(type 3, 99 leaves, 6 steps):

$$
\frac{3(-a d+b c)^{2} \operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{b x^{2}+a}}{\sqrt{b} \sqrt{d x^{2}+c}}\right)}{8 d^{5 / 2} \sqrt{b}}+\frac{\left(b x^{2}+a\right)^{3 / 2} \sqrt{d x^{2}+c}}{4 d}-\frac{3(-a d+b c) \sqrt{b x^{2}+a} \sqrt{d x^{2}+c}}{8 d^{2}}
$$

Result(type 3, 336 leaves):
$\frac{1}{16 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} d^{2} \sqrt{b d}}\left(\sqrt{b x^{2}+a} \sqrt{d x^{2}+c}\left(4 b \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} x^{2} d \sqrt{b d}\right.\right.$
$+3 \ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) a^{2} d^{2}-6 b \ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) a c d$
$+3 b^{2} \ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) c^{2}+10 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} a d \sqrt{b d}$
$\left.\left.-6 b \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} c \sqrt{b d}\right)\right)$

Problem 248: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{3}\left(b x^{2}+a\right)^{5 / 2}}{\sqrt{d x^{2}+c}} \mathrm{~d} x
$$

Optimal(type 3, 199 leaves, 8 steps):


$$
+\frac{\left(b x^{2}+a\right)^{7 / 2} \sqrt{d x^{2}+c}}{8 b d}-\frac{5(-a d+b c)^{2}(a d+7 b c) \sqrt{b x^{2}+a} \sqrt{d x^{2}+c}}{128 b d^{4}}
$$

Result(type 3, 769 leaves):

$$
\begin{aligned}
& -\frac{1}{768 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} d^{4} b \sqrt{b d}}\left(\sqrt { b x ^ { 2 } + a } \sqrt { d x ^ { 2 } + c } \left(-96 x^{6} b^{3} d^{3} \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}\right.\right. \\
& -272 x^{4} a b^{2} d^{3} \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+112 x^{4} b^{3} c d^{2} \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}-236 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} x^{2} a^{2} d^{3} b \sqrt{b d} \\
& \\
& +344 b^{2} \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} x^{2} a c d^{2} \sqrt{b d}-140 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} x^{2} c^{2} b^{3} d \sqrt{b d} \\
& \\
& +15 \ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) a^{4} d^{4}+60 \ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) a^{3} c d^{3} b \\
& \\
& -270 b^{2} \ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) a^{2} c^{2} d^{2} \\
& \\
& +300 \ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) a c^{3} b^{3} d \\
& \\
& -105 b^{4} \ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) c^{4}-30 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} a^{3} d^{3} \sqrt{b d} \\
& \\
& \left.+382 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} a^{2} c d^{2} b \sqrt{b d}-530 b^{2} \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} a c^{2} d \sqrt{b d}+210 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} c^{3} b^{3} \sqrt{b d}\right)
\end{aligned}
$$

Problem 249: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(b x^{2}+a\right)^{5 / 2}}{x \sqrt{d x^{2}+c}} \mathrm{~d} x
$$

Optimal(type 3, 149 leaves, 9 steps):

$$
\begin{aligned}
& \frac{\left(15 a^{2} d^{2}-10 a c b d+3 b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{b x^{2}+a}}{\sqrt{b} \sqrt{d x^{2}+c}}\right) \sqrt{b}}{8 d^{5 / 2}}-\frac{a^{5 / 2} \operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{b x^{2}+a}}{\sqrt{a} \sqrt{d x^{2}+c}}\right)}{\sqrt{c}}+\frac{b\left(b x^{2}+a\right)^{3 / 2} \sqrt{d x^{2}+c}}{4 d} \\
& \quad-\frac{b(-7 a d+3 b c) \sqrt{b x^{2}+a} \sqrt{d x^{2}+c}}{8 d^{2}}
\end{aligned}
$$

Result(type 3, 445 leaves):

$$
\begin{aligned}
& -\frac{1}{16 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} d^{2} \sqrt{b d} \sqrt{a c}}\left(\sqrt { b x ^ { 2 } + a } \sqrt { d x ^ { 2 } + c } \left(-4 b^{2} \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} x^{2} d \sqrt{b d} \sqrt{a c}\right.\right. \\
& \quad+8 a^{3} \ln \left(\frac{x^{2} a d+b c x^{2}+2 \sqrt{a c} \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c}+2 a c}{x^{2}}\right) d^{2} \sqrt{b d} \\
& \quad-15 b \ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) a^{2} d^{2} \sqrt{a c} \\
& \quad+10 b^{2} \ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) a c d \sqrt{a c} \\
& \quad-3 b^{3} \ln \left(\frac{2 b d x^{2}+2 \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) c^{2} \sqrt{a c}-18 b \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} a d \sqrt{b d} \sqrt{a c} \\
& \\
& \left.\left.+6 b^{2} \sqrt{x^{4} b d+x^{2} a d+b c x^{2}+a c} c \sqrt{b d} \sqrt{a c}\right)\right)
\end{aligned}
$$

Problem 262: Unable to integrate problem.

$$
\int \frac{x^{5}}{\left(-x^{2}+1\right)^{1 / 3}\left(x^{2}+3\right)} \mathrm{d} x
$$

Optimal(type 3, 80 leaves, 7 steps):

$$
\frac{3\left(-x^{2}+1\right)^{2 / 3}}{2}+\frac{3\left(-x^{2}+1\right)^{5 / 3}}{10}-\frac{9 \ln \left(x^{2}+3\right) 2^{1 / 3}}{8}+\frac{27 \ln \left(2^{2 / 3}-\left(-x^{2}+1\right)^{1 / 3}\right) 2^{1 / 3}}{8}+\frac{9 \arctan \left(\frac{\left(1+\left(-2 x^{2}+2\right)^{1 / 3}\right) \sqrt{3}}{3}\right) \sqrt{3} 2^{1 / 3}}{4}
$$

Result(type 8, 43 leaves):

$$
\frac{3\left(x^{2}-6\right)\left(x^{2}-1\right)}{10\left(-x^{2}+1\right)^{1 / 3}}+\int \frac{9 x}{\left(x^{2}+3\right)\left(-x^{2}+1\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 263: Unable to integrate problem.

$$
\int \frac{1}{x^{3}\left(-x^{2}+1\right)^{1 / 3}\left(x^{2}+3\right)} \mathrm{d} x
$$

Optimal(type 3, 72 leaves, 7 steps):

$$
-\frac{\left(-x^{2}+1\right)^{2 / 3}}{6 x^{2}}-\frac{\ln \left(x^{2}+3\right) 2^{1 / 3}}{72}+\frac{\ln \left(2^{2 / 3}-\left(-x^{2}+1\right)^{1 / 3}\right) 2^{1 / 3}}{24}+\frac{\arctan \left(\frac{\left(1+\left(-2 x^{2}+2\right)^{1 / 3}\right) \sqrt{3}}{3}\right) \sqrt{3} 2^{1 / 3}}{36}
$$

Result(type 8, 41 leaves):

$$
\frac{x^{2}-1}{6 x^{2}\left(-x^{2}+1\right)^{1 / 3}}+\int \frac{x}{9\left(x^{2}+3\right)\left(-x^{2}+1\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 264: Unable to integrate problem.

$$
\int \frac{x^{4}}{\left(-x^{2}+1\right)^{1 / 3}\left(x^{2}+3\right)} \mathrm{d} x
$$

Optimal(type 4, 420 leaves, 7 steps):
$-\frac{3 x\left(-x^{2}+1\right)^{2 / 3}}{7}-\frac{3 \operatorname{arctanh}(x) 2^{1 / 3}}{4}+\frac{9 \operatorname{arctanh}\left(\frac{x}{\left.1+2^{1 / 3}\left(-x^{2}+1\right)^{1 / 3}\right) 2^{1 / 3}}\right.}{4}+\frac{54 x}{7\left(1-\left(-x^{2}+1^{1 / 3}-\sqrt{3}\right)\right.}+\frac{3 \arctan \left(\frac{\sqrt{3}}{x}\right) \sqrt{3} 2^{1 / 3}}{4}$ $+\frac{3 \arctan \left(\frac{\left(1-2^{1 / 3}\left(-x^{2}+1\right)^{1 / 3}\right) \sqrt{3}}{x}\right) \sqrt{3} 2^{1 / 3}}{4}$

$$
\begin{aligned}
& -\frac{183^{3 / 4}\left(1-\left(-x^{2}+1\right)^{1 / 3}\right) \text { EllipticF }\left(\frac{1-\left(-x^{2}+1\right)^{1 / 3}+\sqrt{3}}{1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{2} \sqrt{\frac{1+\left(-x^{2}+1\right)^{1 / 3}+\left(-x^{2}+1\right)^{2 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}}}{7 x \sqrt{\frac{-1+\left(-x^{2}+1\right)^{1 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}}} \\
& +\frac{273^{1 / 4}\left(1-\left(-x^{2}+1\right)^{1 / 3}\right) \text { EllipticE }\left(\frac{1-\left(-x^{2}+1\right)^{1 / 3}+\sqrt{3}}{1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}}, 2 I-I \sqrt{3}\right) \sqrt{\frac{1+\left(-x^{2}+1\right)^{1 / 3}+\left(-x^{2}+1\right)^{2 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}\left(\frac{\sqrt{6}}{2}\right.}+\frac{\sqrt{2}}{2}}{7 x \sqrt{\frac{-1+\left(-x^{2}+1\right)^{1 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}}}
\end{aligned}
$$

Result(type 8, 45 leaves):

$$
\frac{3 x\left(x^{2}-1\right)}{7\left(-x^{2}+1\right)^{1 / 3}}+\int-\frac{9\left(2 x^{2}-1\right)}{7\left(x^{2}+3\right)\left(-x^{2}+1\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 265: Unable to integrate problem.

$$
\int \frac{x^{2}}{\left(-x^{2}+1\right)^{1 / 3}\left(x^{2}+3\right)} \mathrm{d} x
$$

Optimal(type 4, 407 leaves, 6 steps):
$\frac{\operatorname{arctanh}(x) 2^{1 / 3}}{4}-\frac{3 \operatorname{arctanh}\left(\frac{x}{\left.1+2^{1 / 3}\left(-x^{2}+1\right)^{1 / 3}\right) 2^{1 / 3}}\right.}{4}-\frac{3 x}{1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}}-\frac{\arctan \left(\frac{\sqrt{3}}{x}\right) \sqrt{3} 2^{1 / 3}}{4}$
$-\frac{\arctan \left(\frac{\left(1-2^{1 / 3}\left(-x^{2}+1\right)^{1 / 3}\right) \sqrt{3}}{x}\right) \sqrt{3} 2^{1 / 3}}{4}$

$$
\begin{aligned}
& +\frac{3^{3 / 4}\left(1-\left(-x^{2}+1\right)^{1 / 3}\right) \text { EllipticF }\left(\frac{1-\left(-x^{2}+1\right)^{1 / 3}+\sqrt{3}}{1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{2} \sqrt{\frac{1+\left(-x^{2}+1\right)^{1 / 3}+\left(-x^{2}+1\right)^{2 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}}}{x \sqrt{\frac{-1+\left(-x^{2}+1\right)^{1 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}}} \\
& -\frac{23^{1 / 4}\left(1-\left(-x^{2}+1\right)^{1 / 3}\right) \text { EllipticE }\left(\frac{1-\left(-x^{2}+1\right)^{1 / 3}+\sqrt{3}}{1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}}, 2 I-I \sqrt{3}\right) \sqrt{\frac{1+\left(-x^{2}+1\right)^{1 / 3}+\left(-x^{2}+1\right)^{2 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}\left(\frac{\sqrt{6}}{2}\right.}+\frac{\sqrt{2}}{2}}{2 x \sqrt{\frac{-1+\left(-x^{2}+1\right)^{1 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}}}
\end{aligned}
$$

Result(type 8, 22 leaves):

$$
\int \frac{x^{2}}{\left(-x^{2}+1\right)^{1 / 3}\left(x^{2}+3\right)} \mathrm{d} x
$$

Problem 266: Unable to integrate problem.

$$
\int \frac{1}{x^{3}\left(-x^{2}+1\right)^{1 / 3}\left(x^{2}+3\right)^{2}} d x
$$

Optimal(type 3, 141 leaves, 12 steps):

$$
\begin{aligned}
& -\frac{5\left(-x^{2}+1\right)^{2 / 3}}{72\left(x^{2}+3\right)}-\frac{\left(-x^{2}+1\right)^{2 / 3}}{6 x^{2}\left(x^{2}+3\right)}+\frac{\ln (x)}{54}-\frac{\ln \left(x^{2}+3\right) 2^{1 / 3}}{96}-\frac{\ln \left(1-\left(-x^{2}+1\right)^{1 / 3}\right)}{36}+\frac{\ln \left(2^{2 / 3}-\left(-x^{2}+1\right)^{1 / 3}\right) 2^{1 / 3}}{32} \\
& \quad+\frac{\arctan \left(\frac{\left(1+\left(-2 x^{2}+2\right)^{1 / 3}\right) \sqrt{3}}{3}\right) \sqrt{3} 2^{1 / 3}}{48}-\frac{\arctan \left(\frac{\left(1+2\left(-x^{2}+1\right)^{1 / 3}\right) \sqrt{3}}{3}\right) \sqrt{3}}{54}
\end{aligned}
$$

Result(type 8, 64 leaves):

$$
\frac{\left(x^{2}-1\right)\left(5 x^{2}+12\right)}{72\left(-x^{2}+1\right)^{1 / 3}\left(x^{2}+3\right) x^{2}}+\int \frac{5 x^{2}-12}{108 x\left(x^{2}+3\right)\left(-x^{2}+1\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 267: Unable to integrate problem.

$$
\int \frac{1}{\left(-x^{2}+1\right)^{1 / 3}\left(x^{2}+3\right)^{2}} d x
$$

Optimal (type 4,427 leaves, 7 steps):
$\frac{x\left(-x^{2}+1\right)^{2 / 3}}{24\left(x^{2}+3\right)}-\frac{\operatorname{arctanh}(x) 2^{1 / 3}}{48}+\frac{\operatorname{arctanh}\left(\frac{x}{\left.1+2^{1 / 3}\left(-x^{2}+1\right)^{1 / 3}\right) 2^{1 / 3}}\right.}{16}-\frac{x}{24\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)}+\frac{\arctan \left(\frac{\sqrt{3}}{x}\right) \sqrt{3} 2^{1 / 3}}{48}$
$+\frac{\arctan \left(\frac{\left(1-2^{1 / 3}\left(-x^{2}+1\right)^{1 / 3}\right) \sqrt{3}}{x}\right) \sqrt{3} 2^{1 / 3}}{48}$

$$
\begin{aligned}
& +\frac{3^{3 / 4}\left(1-\left(-x^{2}+1\right)^{1 / 3}\right) \text { EllipticF }\left(\frac{1-\left(-x^{2}+1\right)^{1 / 3}+\sqrt{3}}{1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{2} \sqrt{\frac{1+\left(-x^{2}+1\right)^{1 / 3}+\left(-x^{2}+1\right)^{2 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}}}{72 x \sqrt{\frac{-1+\left(-x^{2}+1\right)^{1 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}}} \\
& -\frac{\left.3^{1 / 4}\left(1-\left(-x^{2}+1\right)^{1 / 3}\right) \text { EllipticE }\left(\frac{1-\left(-x^{2}+1\right)^{1 / 3}+\sqrt{3}}{1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{1+\left(-x^{2}+1\right)^{1 / 3}+\left(-x^{2}+1\right)^{2 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right.}\right)}{48 x \sqrt{\frac{-1+\left(-x^{2}+1\right)^{1 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}}}
\end{aligned}
$$

Result(type 8, 50 leaves):

$$
-\frac{x\left(x^{2}-1\right)}{24\left(x^{2}+3\right)\left(-x^{2}+1\right)^{1 / 3}}+\int \frac{x^{2}+21}{72\left(x^{2}+3\right)\left(-x^{2}+1\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 268: Unable to integrate problem.

$$
\int \frac{1}{x^{2}\left(-x^{2}+1\right)^{1 / 3}\left(x^{2}+3\right)^{2}} d x
$$

Optimal(type 4, 443 leaves, 8 steps):
$-\frac{\left(-x^{2}+1\right)^{2 / 3}}{8 x}+\frac{\left(-x^{2}+1\right)^{2 / 3}}{24 x\left(x^{2}+3\right)}+\frac{7 \operatorname{arctanh}(x) 2^{1 / 3}}{432}-\frac{7 \operatorname{arctanh}\left(\frac{x}{\left.1+2^{1 / 3}\left(-x^{2}+1\right)^{1 / 3}\right) 2^{1 / 3}}\right.}{144}+\frac{x}{8\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)}$

$$
-\frac{7 \arctan \left(\frac{\sqrt{3}}{x}\right) \sqrt{3} 2^{1 / 3}}{432}-\frac{7 \arctan \left(\frac{\left(1-2^{1 / 3}\left(-x^{2}+1\right)^{1 / 3}\right) \sqrt{3}}{x}\right) \sqrt{3} 2^{1 / 3}}{432}
$$

$$
\begin{aligned}
& -\frac{3^{3 / 4}\left(1-\left(-x^{2}+1\right)^{1 / 3}\right) \text { EllipticF }\left(\frac{1-\left(-x^{2}+1\right)^{1 / 3}+\sqrt{3}}{1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{2} \sqrt{\frac{1+\left(-x^{2}+1\right)^{1 / 3}+\left(-x^{2}+1\right)^{2 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}}}{24 x \sqrt{\frac{-1+\left(-x^{2}+1\right)^{1 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}}} \\
& +\frac{3^{1 / 4}\left(1-\left(-x^{2}+1\right)^{1 / 3}\right) \operatorname{EllipticE}\left(\frac{1-\left(-x^{2}+1\right)^{1 / 3}+\sqrt{3}}{1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right) \sqrt{\frac{1+\left(-x^{2}+1\right)^{1 / 3}+\left(-x^{2}+1\right)^{2 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}}{16 x \sqrt{\frac{-1+\left(-x^{2}+1\right)^{1 / 3}}{\left(1-\left(-x^{2}+1\right)^{1 / 3}-\sqrt{3}\right)^{2}}}}
\end{aligned}
$$

Result(type 8, 61 leaves):

$$
\frac{\left(x^{2}-1\right)\left(3 x^{2}+8\right)}{24\left(-x^{2}+1\right)^{1 / 3}\left(x^{2}+3\right) x}+\int-\frac{3 x^{2}+23}{72\left(x^{2}+3\right)\left(-x^{2}+1\right)^{1 / 3}} \mathrm{~d} x
$$

Problem 269: Unable to integrate problem.

$$
\int \frac{x^{4}}{\left(-3 x^{2}+2\right)^{1 / 4}\left(-3 x^{2}+4\right)} \mathrm{d} x
$$

Optimal(type 4, 148 leaves, 6 steps):


$$
45 \cos \left(\frac{\arcsin \left(\frac{x \sqrt{6}}{2}\right)}{2}\right)
$$

Result(type 8, 51 leaves):

$$
-\frac{2\left(3 x^{2}-2\right) x}{45\left(-3 x^{2}+2\right)^{1 / 4}}-\left(\int \frac{8\left(9 x^{2}-2\right)}{45\left(3 x^{2}-4\right)\left(-3 x^{2}+2\right)^{1 / 4}} \mathrm{~d} x\right)
$$

Problem 270: Unable to integrate problem.

$$
\int \frac{x}{\left(3 x^{2}-2\right)\left(3 x^{2}-1\right)^{1 / 4}} d x
$$

Optimal(type 3, 25 leaves, 5 steps):

$$
\frac{\arctan \left(\left(3 x^{2}-1\right)^{1 / 4}\right)}{3}-\frac{\operatorname{arctanh}\left(\left(3 x^{2}-1\right)^{1 / 4}\right)}{3}
$$

$$
\int \frac{x}{\left(3 x^{2}-2\right)\left(3 x^{2}-1\right)^{1 / 4}} \mathrm{~d} x
$$

Problem 271: Unable to integrate problem.

$$
\int \frac{1}{\left(3 x^{2}-2\right)\left(3 x^{2}-1\right)^{1 / 4}} \mathrm{~d} x
$$

Optimal(type 3, 43 leaves, 1 step):

$$
-\frac{\arctan \left(\frac{x \sqrt{6}}{2\left(3 x^{2}-1\right)^{1 / 4}}\right) \sqrt{6}}{12}-\frac{\operatorname{arctanh}\left(\frac{x \sqrt{6}}{2\left(3 x^{2}-1\right)^{1 / 4}}\right) \sqrt{6}}{12}
$$

Result(type 8, 21 leaves):

$$
\int \frac{1}{\left(3 x^{2}-2\right)\left(3 x^{2}-1\right)^{1 / 4}} \mathrm{~d} x
$$

Problem 272: Unable to integrate problem.

$$
\int \frac{x^{2}}{\left(3 x^{2}+2\right)^{3 / 4}\left(3 x^{2}+4\right)} d x
$$

Optimal(type 3, 93 leaves, 1 step):

$$
-\frac{\arctan \left(\frac{\left(22^{3 / 4}+22^{1 / 4} \sqrt{3 x^{2}+2}\right) \sqrt{3}}{6 x\left(3 x^{2}+2\right)^{1 / 4}}\right) 2^{3 / 4} \sqrt{3}}{18}+\frac{\operatorname{arctanh}\left(\frac{\left(22^{3 / 4}-22^{1 / 4} \sqrt{3 x^{2}+2}\right) \sqrt{3}}{6 x\left(3 x^{2}+2\right)^{1 / 4}}\right) 2^{3 / 4} \sqrt{3}}{18}
$$

Result(type 8, 24 leaves):

$$
\int \frac{x^{2}}{\left(3 x^{2}+2\right)^{3 / 4}\left(3 x^{2}+4\right)} \mathrm{d} x
$$

Problem 273: Unable to integrate problem.

$$
\int \frac{x^{2}}{\left(3 x^{2}+a\right)^{3 / 4}\left(3 x^{2}+2 a\right)} \mathrm{d} x
$$

Optimal(type 3, 90 leaves, 1 step):

$$
-\frac{\arctan \left(\frac{a^{3 / 4}\left(1+\frac{\sqrt{3 x^{2}+a}}{\sqrt{a}}\right) \sqrt{3}}{3 x\left(3 x^{2}+a\right)^{1 / 4}}\right) \sqrt{3}}{9 a^{1 / 4}}+\frac{\operatorname{arctanh}\left(\frac{a^{3 / 4}\left(1-\frac{\sqrt{3 x^{2}+a}}{\sqrt{a}}\right) \sqrt{3}}{3 x\left(3 x^{2}+a\right)^{1 / 4}}\right) \sqrt{3}}{9 a^{1 / 4}}
$$

Result(type 8, 26 leaves):

$$
\int \frac{x^{2}}{\left(3 x^{2}+a\right)^{3 / 4}\left(3 x^{2}+2 a\right)} \mathrm{d} x
$$

Problem 274: Unable to integrate problem.

$$
\int \frac{x^{2}}{\left(b x^{2}+a\right)^{3 / 4}\left(b x^{2}+2 a\right)} \mathrm{d} x
$$

Optimal(type 3, 87 leaves, 1 step):

$$
-\frac{\arctan \left(\frac{a^{3 / 4}\left(1+\frac{\sqrt{b x^{2}+a}}{\sqrt{a}}\right)}{x\left(b x^{2}+a\right)^{1 / 4} \sqrt{b}}\right)}{a^{1 / 4} b^{3 / 2}}+\frac{\operatorname{arctanh}\left(\frac{a^{3 / 4}\left(1-\frac{\sqrt{b x^{2}+a}}{\sqrt{a}}\right)}{x\left(b x^{2}+a\right)^{1 / 4} \sqrt{b}}\right)}{a^{1 / 4} b^{3 / 2}}
$$

Result(type 8, 26 leaves):

$$
\int \frac{x^{2}}{\left(b x^{2}+a\right)^{3 / 4}\left(b x^{2}+2 a\right)} \mathrm{d} x
$$

Problem 275: Unable to integrate problem.

$$
\int \frac{x^{2}}{\left(-b x^{2}+a\right)^{3 / 4}\left(-b x^{2}+2 a\right)} \mathrm{d} x
$$

Optimal(type 3, 91 leaves, 1 step):

$$
\frac{\arctan \left(\frac{a^{3 / 4}\left(1-\frac{\sqrt{-b x^{2}+a}}{\sqrt{a}}\right)}{x\left(-b x^{2}+a\right)^{1 / 4} \sqrt{b}}\right)}{a^{1 / 4} b^{3 / 2}}-\frac{\operatorname{arctanh}\left(\frac{a^{3 / 4}\left(1+\frac{\sqrt{-b x^{2}+a}}{\sqrt{a}}\right)}{x\left(-b x^{2}+a\right)^{1 / 4} \sqrt{b}}\right)}{a^{1 / 4} b^{3 / 2}}
$$

Result(type 8, 28 leaves):

$$
\int \frac{x^{2}}{\left(-b x^{2}+a\right)^{3 / 4}\left(-b x^{2}+2 a\right)} \mathrm{d} x
$$

Problem 276: Unable to integrate problem.

$$
\int \frac{x^{3}}{\left(-3 x^{2}+2\right)^{3 / 4}\left(-3 x^{2}+4\right)} \mathrm{d} x
$$

Optimal(type 3, 115 leaves, 14 steps):

$$
\begin{aligned}
& \frac{2\left(-3 x^{2}+2\right)^{1 / 4}}{9}-\frac{2^{3 / 4} \arctan \left(1+\left(-6 x^{2}+4\right)^{1 / 4}\right)}{9}-\frac{2^{3 / 4} \arctan \left(2^{1 / 4}\left(-3 x^{2}+2\right)^{1 / 4}-1\right)}{9} \\
& \quad+\frac{\ln \left(-2^{3 / 4}\left(-3 x^{2}+2\right)^{1 / 4}+\sqrt{2}+\sqrt{-3 x^{2}+2}\right) 2^{3 / 4}}{18}-\frac{\ln \left(2^{3 / 4}\left(-3 x^{2}+2\right)^{1 / 4}+\sqrt{2}+\sqrt{-3 x^{2}+2}\right) 2^{3 / 4}}{18}
\end{aligned}
$$

Result(type 8, 70 leaves):

$$
-\frac{2\left(3 x^{2}-2\right)}{9\left(-3 x^{2}+2\right)^{3 / 4}}-\frac{\left(\int \frac{4 x}{3\left(3 x^{2}-4\right)\left(-\left(3 x^{2}-2\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(-\left(3 x^{2}-2\right)^{3}\right)^{1 / 4}}{\left(-3 x^{2}+2\right)^{3 / 4}}
$$

Problem 277: Unable to integrate problem.

$$
\int \frac{x^{6}}{\left(-3 x^{2}+2\right)^{3 / 4}\left(-3 x^{2}+4\right)} \mathrm{d} x
$$

Optimal(type 4, 162 leaves, 11 steps):
$\frac{80 x\left(-3 x^{2}+2\right)^{1 / 4}}{567}+\frac{2 x^{3}\left(-3 x^{2}+2\right)^{1 / 4}}{63}+\frac{82^{3 / 4} \arctan \left(\frac{\left(2^{3 / 4}-2^{1 / 4} \sqrt{-3 x^{2}+2}\right) \sqrt{3}}{3 x\left(-3 x^{2}+2\right)^{1 / 4}}\right) \sqrt{3}}{81}$
$-\frac{82^{3 / 4} \operatorname{arctanh}\left(\frac{\left(2^{3 / 4}+2^{1 / 4} \sqrt{-3 x^{2}+2}\right) \sqrt{3}}{3 x\left(-3 x^{2}+2\right)^{1 / 4}}\right) \sqrt{3}}{81}-\frac{1602^{3 / 4} \sqrt{\cos \left(\frac{\arcsin \left(\frac{x \sqrt{6}}{2}\right)^{2}}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\arcsin \left(\frac{x \sqrt{6}}{2}\right)}{2}\right), \sqrt{2}\right) \sqrt{3}}{1701 \cos \left(\frac{\arcsin \left(\frac{x \sqrt{6}}{2}\right)}{2}\right)}$
Result(type 8, 84 leaves):

$$
-\frac{2 x\left(9 x^{2}+40\right)\left(3 x^{2}-2\right)}{567\left(-3 x^{2}+2\right)^{3 / 4}}-\frac{\left(\int \frac{16\left(93 x^{2}-40\right)}{567\left(3 x^{2}-4\right)\left(-\left(3 x^{2}-2\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(-\left(3 x^{2}-2\right)^{3}\right)^{1 / 4}}{\left(-3 x^{2}+2\right)^{3 / 4}}
$$

Problem 278: Unable to integrate problem.

$$
\int \frac{x^{2}}{\left(3 x^{2}-2\right)\left(3 x^{2}-1\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 3, 43 leaves, 1 step):

$$
\frac{\arctan \left(\frac{x \sqrt{6}}{2\left(3 x^{2}-1\right)^{1 / 4}}\right) \sqrt{6}}{18}-\frac{\operatorname{arctanh}\left(\frac{x \sqrt{6}}{2\left(3 x^{2}-1\right)^{1 / 4}}\right) \sqrt{6}}{18}
$$

Result (type 8, 24 leaves):

$$
\int \frac{x^{2}}{\left(3 x^{2}-2\right)\left(3 x^{2}-1\right)^{3 / 4}} \mathrm{~d} x
$$

Problem 279: Unable to integrate problem.

$$
\int \frac{x^{2}}{\left(b x^{2}-2\right)\left(b x^{2}-1\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 3, 55 leaves, 1 step):

$$
\frac{\arctan \left(\frac{x \sqrt{b} \sqrt{2}}{2\left(b x^{2}-1\right)^{1 / 4}}\right) \sqrt{2}}{2 b^{3 / 2}}-\frac{\operatorname{arctanh}\left(\frac{x \sqrt{b} \sqrt{2}}{2\left(b x^{2}-1\right)^{1 / 4}}\right) \sqrt{2}}{2 b^{3 / 2}}
$$

Result(type 8, 24 leaves):

$$
\int \frac{x^{2}}{\left(b x^{2}-2\right)\left(b x^{2}-1\right)^{3 / 4}} \mathrm{~d} x
$$

Problem 280: Unable to integrate problem.

$$
\int \frac{x^{2}}{\left(-b x^{2}-2\right)\left(-b x^{2}-1\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal (type 3, 57 leaves, 1 step):

$$
\frac{\arctan \left(\frac{x \sqrt{b} \sqrt{2}}{2\left(-b x^{2}-1\right)^{1 / 4}}\right) \sqrt{2}}{2 b^{3 / 2}}-\frac{\operatorname{arctanh}\left(\frac{x \sqrt{b} \sqrt{2}}{2\left(-b x^{2}-1\right)^{1 / 4}}\right) \sqrt{2}}{2 b^{3 / 2}}
$$

Result(type 8, 26 leaves):

$$
\int \frac{x^{2}}{\left(-b x^{2}-2\right)\left(-b x^{2}-1\right)^{3 / 4}} \mathrm{~d} x
$$

Problem 281: Unable to integrate problem.

$$
\int \frac{x^{7}}{\left(3 x^{2}-2\right)\left(3 x^{2}-1\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 3, 58 leaves, 7 steps):

$$
\frac{14\left(3 x^{2}-1\right)^{1 / 4}}{81}+\frac{8\left(3 x^{2}-1\right)^{5 / 4}}{405}+\frac{2\left(3 x^{2}-1\right)^{9 / 4}}{729}-\frac{8 \arctan \left(\left(3 x^{2}-1\right)^{1 / 4}\right)}{81}-\frac{8 \operatorname{arctanh}\left(\left(3 x^{2}-1\right)^{1 / 4}\right)}{81}
$$

Result(type 8, 70 leaves):

$$
\frac{2\left(45 x^{4}+78 x^{2}+284\right)\left(3 x^{2}-1\right)^{1 / 4}}{3645}+\frac{\left(\int \frac{8 x}{27\left(3 x^{2}-2\right)\left(\left(3 x^{2}-1\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(3 x^{2}-1\right)^{3}\right)^{1 / 4}}{\left(3 x^{2}-1\right)^{3 / 4}}
$$

Problem 282: Unable to integrate problem.

$$
\int \frac{x^{3}}{\left(3 x^{2}-2\right)\left(3 x^{2}-1\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 3, 36 leaves, 6 steps):

$$
\frac{2\left(3 x^{2}-1\right)^{1 / 4}}{9}-\frac{2 \arctan \left(\left(3 x^{2}-1\right)^{1 / 4}\right)}{9}-\frac{2 \operatorname{arctanh}\left(\left(3 x^{2}-1\right)^{1 / 4}\right)}{9}
$$

Result(type 8, 58 leaves):

$$
\frac{2\left(3 x^{2}-1\right)^{1 / 4}}{9}+\frac{\left(\int \frac{2 x}{3\left(3 x^{2}-2\right)\left(\left(3 x^{2}-1\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(3 x^{2}-1\right)^{3}\right)^{1 / 4}}{\left(3 x^{2}-1\right)^{3 / 4}}
$$

Problem 283: Unable to integrate problem.

$$
\int \frac{x^{6}}{\left(3 x^{2}-2\right)\left(3 x^{2}-1\right)^{3 / 4}} d x
$$

Optimal(type 4, 158 leaves, 15 steps):
$\frac{40 x\left(3 x^{2}-1\right)^{1 / 4}}{567}+\frac{2 x^{3}\left(3 x^{2}-1\right)^{1 / 4}}{63}+\frac{2 \arctan \left(\frac{x \sqrt{6}}{2\left(3 x^{2}-1\right)^{1 / 4}}\right) \sqrt{6}}{81}-\frac{2 \operatorname{arctanh}\left(\frac{x \sqrt{6}}{2\left(3 x^{2}-1\right)^{1 / 4}}\right) \sqrt{6}}{81}$
$+\frac{40 \sqrt{\cos \left(2 \arctan \left(\left(3 x^{2}-1\right)^{1 / 4}\right)\right)^{2}} \operatorname{EllipticF}\left(\sin \left(2 \arctan \left(\left(3 x^{2}-1\right)^{1 / 4}\right)\right), \frac{\sqrt{2}}{2}\right)\left(1+\sqrt{3 x^{2}-1}\right) \sqrt{\frac{x^{2}}{\left(1+\sqrt{3 x^{2}-1}\right)^{2}} \sqrt{3}}}{1701 \cos \left(2 \arctan \left(\left(3 x^{2}-1\right)^{1 / 4}\right)\right) x}$
Result(type 8, 72 leaves):

$$
\frac{2 x\left(9 x^{2}+20\right)\left(3 x^{2}-1\right)^{1 / 4}}{567}+\frac{\left(\int \frac{4\left(93 x^{2}-20\right)}{567\left(3 x^{2}-2\right)\left(\left(3 x^{2}-1\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(3 x^{2}-1\right)^{3}\right)^{1 / 4}}{\left(3 x^{2}-1\right)^{3 / 4}}
$$

Problem 284: Unable to integrate problem.

$$
\int \frac{x^{4}}{\left(3 x^{2}-2\right)\left(3 x^{2}-1\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 144 leaves, 11 steps):
$\frac{2 x\left(3 x^{2}-1\right)^{1 / 4}}{27}+\frac{\arctan \left(\frac{x \sqrt{6}}{2\left(3 x^{2}-1\right)^{1 / 4}}\right) \sqrt{6}}{27}-\frac{\operatorname{arctanh}\left(\frac{x \sqrt{6}}{2\left(3 x^{2}-1\right)^{1 / 4}}\right) \sqrt{6}}{27}$
$+\frac{2 \sqrt{\cos \left(2 \arctan \left(\left(3 x^{2}-1\right)^{1 / 4}\right)\right)^{2}} \text { EllipticF }\left(\sin \left(2 \arctan \left(\left(3 x^{2}-1\right)^{1 / 4}\right)\right), \frac{\sqrt{2}}{2}\right)\left(1+\sqrt{3 x^{2}-1}\right) \sqrt{\frac{x^{2}}{\left(1+\sqrt{3 x^{2}-1}\right)^{2}} \sqrt{3}}}{81 \cos \left(2 \arctan \left(\left(3 x^{2}-1\right)^{1 / 4}\right)\right) x}$
Result(type 8, 65 leaves):

$$
\frac{2 x\left(3 x^{2}-1\right)^{1 / 4}}{27}+\frac{\left(\int \frac{4\left(6 x^{2}-1\right)}{27\left(3 x^{2}-2\right)\left(\left(3 x^{2}-1\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(3 x^{2}-1\right)^{3}\right)^{1 / 4}}{\left(3 x^{2}-1\right)^{3 / 4}}
$$

Problem 285: Unable to integrate problem.

$$
\int \frac{x^{2}}{\left(3 x^{2}-2\right)\left(3 x^{2}-1\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 3, 43 leaves, 1 step):

$$
\frac{\arctan \left(\frac{x \sqrt{6}}{2\left(3 x^{2}-1\right)^{1 / 4}}\right) \sqrt{6}}{18}-\frac{\operatorname{arctanh}\left(\frac{x \sqrt{6}}{2\left(3 x^{2}-1\right)^{1 / 4}}\right) \sqrt{6}}{18}
$$

Result(type 8, 24 leaves):

$$
\int \frac{x^{2}}{\left(3 x^{2}-2\right)\left(3 x^{2}-1\right)^{3 / 4}} \mathrm{~d} x
$$

Problem 286: Unable to integrate problem.

$$
\int \frac{(e x)^{5 / 2}\left(d x^{2}+c\right)}{\left(b x^{2}+a\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal (type 3, 133 leaves, 7 steps):
$\frac{(-7 a d+8 b c) e(e x)^{3 / 2}\left(b x^{2}+a\right)^{1 / 4}}{16 b^{2}}+\frac{d(e x)^{7 / 2}\left(b x^{2}+a\right)^{1 / 4}}{4 b e}+\frac{3 a(-7 a d+8 b c) e^{5 / 2} \arctan \left(\frac{b^{1 / 4} \sqrt{e x}}{\left(b x^{2}+a\right)^{1 / 4} \sqrt{e}}\right)}{32 b^{11 / 4}}$

$$
-\frac{3 a(-7 a d+8 b c) e^{5 / 2} \operatorname{arctanh}\left(\frac{b^{1 / 4} \sqrt{e x}}{\left(b x^{2}+a\right)^{1 / 4} \sqrt{e}}\right)}{32 b^{11 / 4}}
$$

Result(type 8, 115 leaves):

$$
-\frac{x\left(-4 b d x^{2}+7 a d-8 b c\right)\left(b x^{2}+a\right)^{1 / 4} e^{2} \sqrt{e x}}{16 b^{2}}+\frac{\left(\int \frac{3 a(7 a d-8 b c) x}{32 b^{2}\left(e^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}} \mathrm{dx}\right) e^{2} \sqrt{e x}\left(e^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}}{x\left(b x^{2}+a\right)^{3 / 4}}
$$

Problem 287: Unable to integrate problem.

$$
\int \frac{(e x)^{7 / 2}\left(d x^{2}+c\right)}{\left(b x^{2}+a\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 177 leaves, 8 steps):
$\frac{(-9 a d+10 b c) e(e x)^{5 / 2}\left(b x^{2}+a\right)^{1 / 4}}{30 b^{2}}+\frac{d(e x)^{9 / 2}\left(b x^{2}+a\right)^{1 / 4}}{5 b e}$

$$
-\frac{a^{3 / 2}(-9 a d+10 b c) e^{2}\left(1+\frac{a}{b x^{2}}\right)^{3 / 4}(e x)^{3 / 2} \sqrt{\cos \left(\frac{\left.\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)\right)^{2}}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}{12 \cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) b^{5 / 2}\left(b x^{2}+a\right)^{3 / 4}}
$$

$$
-\frac{a(-9 a d+10 b c) e^{3}\left(b x^{2}+a\right)^{1 / 4} \sqrt{e x}}{12 b^{3}}
$$

Result(type 8, 137 leaves):

$$
\frac{\left(12 b^{2} d x^{4}-18 a b d x^{2}+20 b^{2} c x^{2}+45 a^{2} d-50 a b c\right)\left(b x^{2}+a\right)^{1 / 4} e^{3} \sqrt{e x}}{60 b^{3}}+\frac{\left(\int-\frac{a^{2}(9 a d-10 b c)}{24 b^{3}\left(e^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}} \mathrm{dx}\right) e^{3} \sqrt{e x}\left(e^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}}{x\left(b x^{2}+a\right)^{3 / 4}}
$$

Problem 288: Unable to integrate problem.

$$
\int \frac{(e x)^{3 / 2}\left(d x^{2}+c\right)}{\left(b x^{2}+a\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 142 leaves, 7 steps):
$\frac{d(e x)^{5 / 2}\left(b x^{2}+a\right)^{1 / 4}}{3 b e}+\frac{(-5 a d+6 b c)\left(1+\frac{a}{b x^{2}}\right)^{3 / 4}(e x)^{3 / 2} \sqrt{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)^{2}}{2}\right)} \operatorname{EllipticF}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{a}}{6 \cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) b^{3 / 2}\left(b x^{2}+a\right)^{3 / 4}}$

$$
+\frac{(-5 a d+6 b c) e\left(b x^{2}+a\right)^{1 / 4} \sqrt{e x}}{6 b^{2}}
$$

Result(type 8, 109 leaves):

$$
-\frac{\left(-2 b d x^{2}+5 a d-6 b c\right)\left(b x^{2}+a\right)^{1 / 4} e \sqrt{e x}}{6 b^{2}}+\frac{\left(\int \frac{a(5 a d-6 b c)}{12 b^{2}\left(e^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right) e \sqrt{e x}\left(e^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}}{x\left(b x^{2}+a\right)^{3 / 4}}
$$

Problem 289: Unable to integrate problem.

$$
\int \frac{d x^{2}+c}{\sqrt{e x}\left(b x^{2}+a\right)^{3 / 4}} d x
$$

Optimal(type 4, 115 leaves, 6 steps):

$$
-\frac{(-a d+2 b c)\left(1+\frac{a}{b x^{2}}\right)^{3 / 4}(e x)^{3 / 2} \sqrt{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) e^{2}\left(b x^{2}+a\right)^{3 / 4} \sqrt{a} \sqrt{b}}
$$

Result(type 8, 87 leaves):

$$
\frac{d\left(b x^{2}+a\right)^{1 / 4} x}{b \sqrt{e x}}+\frac{\left(\int-\frac{a d-2 b c}{2 b\left(e^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(e^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}}{\sqrt{e x}\left(b x^{2}+a\right)^{3 / 4}}
$$

Problem 290: Unable to integrate problem.

$$
\int \frac{d x^{2}+c}{(e x)^{13 / 2}\left(b x^{2}+a\right)^{3 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 179 leaves, 8 steps):

$$
-\frac{2 c\left(b x^{2}+a\right)^{1 / 4}}{11 a e(e x)^{11 / 2}}+\frac{2(-11 a d+10 b c)\left(b x^{2}+a\right)^{1 / 4}}{77 a^{2} e^{3}(e x)^{7 / 2}}-\frac{4 b(-11 a d+10 b c)\left(b x^{2}+a\right)^{1 / 4}}{77 a^{3} e^{5}(e x)^{3 / 2}}
$$

$$
+\frac{8 b^{5 / 2}(-11 a d+10 b c)\left(1+\frac{a}{b x^{2}}\right)^{3 / 4}(e x)^{3 / 2} \sqrt{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)}{\left(\frac{1}{2}\right)}
$$

$$
77 \cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) a^{7 / 2} e^{8}\left(b x^{2}+a\right)^{3 / 4}
$$

Result(type 8, 140 leaves):

$$
-\frac{2\left(b x^{2}+a\right)^{1 / 4}\left(-22 a b d x^{4}+20 b^{2} c x^{4}+11 a^{2} d x^{2}-10 a b c x^{2}+7 a^{2} c\right)}{77 a^{3} x^{5} e^{6} \sqrt{e x}}+\frac{\left(\int \frac{4 b^{2}(11 a d-10 b c)}{77 a^{3}\left(e^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right)\left(e^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}}{e^{6} \sqrt{e x}\left(b x^{2}+a\right)^{3 / 4}}
$$

Problem 292: Unable to integrate problem.

$$
\int \frac{(e x)^{9 / 2}\left(d x^{2}+c\right)}{\left(b x^{2}+a\right)^{5 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 177 leaves, 6 steps):

$$
-\frac{7 a(-11 a d+10 b c) e^{3}(e x)^{3 / 2}}{60 b^{3}\left(b x^{2}+a\right)^{1 / 4}}+\frac{(-11 a d+10 b c) e(e x)^{7 / 2}}{30 b^{2}\left(b x^{2}+a\right)^{1 / 4}}+\frac{d(e x)^{11 / 2}}{5 b e\left(b x^{2}+a\right)^{1 / 4}}
$$

$$
-\frac{7 a^{3 / 2}(-11 a d+10 b c) e^{4}\left(1+\frac{a}{b x^{2}}\right)^{1 / 4} \sqrt{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{e x}}{20 \cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) b^{7 / 2}\left(b x^{2}+a\right)^{1 / 4}}
$$

Result(type 8, 142 leaves):

$$
-\frac{x\left(-6 b d x^{2}+17 a d-10 b c\right)\left(b x^{2}+a\right)^{3 / 4} e^{4} \sqrt{e x}}{30 b^{3}}+\frac{\left(\int \frac{a x\left(37 a b d x^{2}-30 b^{2} c x^{2}+17 a^{2} d-10 a b c\right)}{20 b^{4}\left(x^{2}+\frac{a}{b}\right)\left(\left(b x^{2}+a\right) e^{2} x^{2}\right)^{1 / 4}} \mathrm{~d} x\right) e^{4} \sqrt{e x}\left(\left(b x^{2}+a\right) e^{2} x^{2}\right)^{1 / 4}}{x\left(b x^{2}+a\right)^{1 / 4}}
$$

Problem 293: Unable to integrate problem.

$$
\int \frac{d x^{2}+c}{(e x)^{3 / 2}\left(b x^{2}+a\right)^{5 / 4}} d x
$$

Optimal(type 4, 116 leaves, 4 steps):

$$
-\frac{2 c}{a e\left(b x^{2}+a\right)^{1 / 4} \sqrt{e x}}+\frac{2(-a d+2 b c)\left(1+\frac{a}{b x^{2}}\right)^{1 / 4} \sqrt{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticE}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{e x}}{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right) a^{3 / 2} e^{2}\left(b x^{2}+a\right)^{1 / 4} \sqrt{b}}
$$

Result(type 8, 114 leaves):

$$
-\frac{2 c\left(b x^{2}+a\right)^{3 / 4}}{a^{2} e \sqrt{e x}}+\frac{\left(\int \frac{x\left(2 b^{2} c x^{2}+a^{2} d+a b c\right)}{a^{2} b\left(x^{2}+\frac{a}{b}\right)\left(\left(b x^{2}+a\right) e^{2} x^{2}\right)^{1 / 4}} \mathrm{~d} x\right)\left(\left(b x^{2}+a\right) e^{2} x^{2}\right)^{1 / 4}}{e \sqrt{e x}\left(b x^{2}+a\right)^{1 / 4}}
$$

Problem 294: Unable to integrate problem.

$$
\int \frac{(e x)^{5 / 2}\left(d x^{2}+c\right)}{\left(b x^{2}+a\right)^{7 / 4}} \mathrm{~d} x
$$

Optimal(type 3, 144 leaves, 7 steps):
$\frac{2(-a d+b c)(e x)^{7 / 2}}{3 a b e\left(b x^{2}+a\right)^{3 / 4}}-\frac{(-7 a d+4 b c) e(e x)^{3 / 2}\left(b x^{2}+a\right)^{1 / 4}}{6 a b^{2}}-\frac{(-7 a d+4 b c) e^{5 / 2} \arctan \left(\frac{b^{1 / 4} \sqrt{e x}}{\left(b x^{2}+a\right)^{1 / 4} \sqrt{e}}\right)}{4 b^{11 / 4}}$

$$
+\frac{(-7 a d+4 b c) e^{5 / 2} \operatorname{arctanh}\left(\frac{b^{1 / 4} \sqrt{e x}}{\left(b x^{2}+a\right)^{1 / 4} \sqrt{e}}\right)}{4 b^{11 / 4}}
$$

Result(type 8, 125 leaves):

$$
\frac{d x\left(b x^{2}+a\right)^{1 / 4} e^{2} \sqrt{e x}}{2 b^{2}}+\frac{\left(\int-\frac{x\left(7 a b d x^{2}-4 b^{2} c x^{2}+3 a^{2} d\right)}{4 b^{3}\left(x^{2}+\frac{a}{b}\right)\left(e^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right) e^{2} \sqrt{e x}\left(e^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}}{x\left(b x^{2}+a\right)^{3 / 4}}
$$

Problem 295: Unable to integrate problem.

$$
\int \frac{\sqrt{e x}\left(d x^{2}+c\right)}{\left(b x^{2}+a\right)^{7 / 4}} \mathrm{~d} x
$$

Optimal(type 3, 95 leaves, 6 steps):

$$
\frac{2(-a d+b c)(e x)^{3 / 2}}{3 a b e\left(b x^{2}+a\right)^{3 / 4}}-\frac{d \arctan \left(\frac{b^{1 / 4} \sqrt{e x}}{\left(b x^{2}+a\right)^{1 / 4} \sqrt{e}}\right) \sqrt{e}}{b^{7 / 4}}+\frac{d \operatorname{arctanh}\left(\frac{b^{1 / 4} \sqrt{e x}}{\left(b x^{2}+a\right)^{1 / 4} \sqrt{e}}\right) \sqrt{e}}{b^{7 / 4}}
$$

Result(type 8, 24 leaves):

$$
\int \frac{\sqrt{e x}\left(d x^{2}+c\right)}{\left(b x^{2}+a\right)^{7 / 4}} \mathrm{~d} x
$$

Problem 297: Unable to integrate problem.

$$
\int \frac{(e x)^{7 / 2}\left(d x^{2}+c\right)}{\left(b x^{2}+a\right)^{7 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 189 leaves, 8 steps):
$\frac{2(-a d+b c)(e x)^{9 / 2}}{3 a b e\left(b x^{2}+a\right)^{3 / 4}}-\frac{(-3 a d+2 b c) e(e x)^{5 / 2}\left(b x^{2}+a\right)^{1 / 4}}{3 a b^{2}}$
$\left.\left.+\frac{5(-3 a d+2 b c) e^{2}\left(1+\frac{a}{b x^{2}}\right)^{3 / 4}(e x)^{3 / 2} \sqrt{\cos \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right)^{2}} \operatorname{EllipticF}\left(\sin \left(\frac{\operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right) \sqrt{a}}{6 \operatorname{arccot}\left(\frac{x \sqrt{b}}{\sqrt{a}}\right)}\right)_{2}^{2}\right) b^{5 / 2}\left(b x^{2}+a\right)^{3 / 4}$

$$
+\frac{5(-3 a d+2 b c) e^{3}\left(b x^{2}+a\right)^{1 / 4} \sqrt{e x}}{6 b^{3}}
$$

Result(type 8, 144 leaves):

$$
-\frac{\left(-2 b d x^{2}+11 a d-6 b c\right)\left(b x^{2}+a\right)^{1 / 4} e^{3} \sqrt{e x}}{6 b^{3}}+\frac{\left(\int \frac{a\left(23 a b d x^{2}-18 b^{2} c x^{2}+11 a^{2} d-6 a b c\right)}{12 b^{4}\left(x^{2}+\frac{a}{b}\right)\left(e^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}} \mathrm{~d} x\right) e^{3} \sqrt{e x}\left(e^{2} x^{2}\left(b x^{2}+a\right)^{3}\right)^{1 / 4}}{x\left(b x^{2}+a\right)^{3 / 4}}
$$

Problem 298: Unable to integrate problem.

$$
\int \frac{(e x)^{13 / 2}\left(d x^{2}+c\right)}{\left(b x^{2}+a\right)^{9 / 4}} \mathrm{~d} x
$$

Optimal(type 4, 221 leaves, 7 steps):
$\frac{2(-a d+b c)(e x)^{15 / 2}}{5 a b e\left(b x^{2}+a\right)^{5 / 4}}-\frac{77 a(-3 a d+2 b c) e^{5}(e x)^{3 / 2}}{60 b^{4}\left(b x^{2}+a\right)^{1 / 4}}+\frac{11(-3 a d+2 b c) e^{3}(e x)^{7 / 2}}{30 b^{3}\left(b x^{2}+a\right)^{1 / 4}}-\frac{(-3 a d+2 b c) e(e x)^{11 / 2}}{5 a b^{2}\left(b x^{2}+a\right)^{1 / 4}}$


Result(type 8, 177 leaves):
$-\frac{x\left(-6 b d x^{2}+27 a d-10 b c\right)\left(b x^{2}+a\right)^{3 / 4} e^{6} \sqrt{e x}}{30 b^{4}}$

$$
+\frac{\left(\int \frac{a x\left(87 a b^{2} d x^{4}-50 b^{3} c x^{4}+94 a^{2} b d x^{2}-40 a b^{2} c x^{2}+27 a^{3} d-10 a^{2} b c\right)}{20 b^{6}\left(x^{4}+\frac{2 a x^{2}}{b}+\frac{a^{2}}{b^{2}}\right)\left(\left(b x^{2}+a\right) e^{2} x^{2}\right)^{1 / 4}} \mathrm{~d} x\right) e^{6} \sqrt{e x}\left(\left(b x^{2}+a\right) e^{2} x^{2}\right)^{1 / 4}}{x\left(b x^{2}+a\right)^{1 / 4}}
$$

Problem 299: Unable to integrate problem.

$$
\int(e x)^{m}\left(b x^{2}+a\right)^{p}\left(d x^{2}+c\right)^{q} \mathrm{~d} x
$$

Optimal(type 6, 97 leaves, 3 steps):

$$
\frac{(e x)^{1+m}\left(b x^{2}+a\right)^{p}\left(d x^{2}+c\right)^{q} \text { AppellF1 }\left(\frac{1}{2}+\frac{m}{2},-p,-q, \frac{3}{2}+\frac{m}{2},-\frac{b x^{2}}{a},-\frac{d x^{2}}{c}\right)}{e(1+m)\left(1+\frac{b x^{2}}{a}\right)^{p}\left(1+\frac{d x^{2}}{c}\right)^{q}}
$$

Result(type 8, 26 leaves):

$$
\int(e x)^{m}\left(b x^{2}+a\right)^{p}\left(d x^{2}+c\right)^{q} \mathrm{~d} x
$$

Problem 300: Unable to integrate problem.

$$
\int\left(b x^{2}+a\right)^{p}\left(d x^{2}+c\right)^{q} \mathrm{~d} x
$$

Optimal(type 6, 75 leaves, 3 steps):

$$
\frac{x\left(b x^{2}+a\right)^{p}\left(d x^{2}+c\right)^{q} \operatorname{AppellF1}\left(\frac{1}{2},-p,-q, \frac{3}{2},-\frac{b x^{2}}{a},-\frac{d x^{2}}{c}\right)}{\left(1+\frac{b x^{2}}{a}\right)^{p}\left(1+\frac{d x^{2}}{c}\right)^{q}}
$$

Result(type 8, 21 leaves):

$$
\int\left(b x^{2}+a\right)^{p}\left(d x^{2}+c\right)^{q} \mathrm{~d} x
$$

Problem 301: Unable to integrate problem.

$$
\int x^{5}\left(b x^{2}+a\right)^{p}\left(d x^{2}+c\right)^{q} \mathrm{~d} x
$$

Optimal(type 5, 238 leaves, 5 steps):
$-\frac{(b c(2+p)+a d(2+q))\left(b x^{2}+a\right)^{1+p}\left(d x^{2}+c\right)^{1+q}}{2 b^{2} d^{2}(2+p+q)(3+p+q)}+\frac{x^{2}\left(b x^{2}+a\right)^{1+p}\left(d x^{2}+c\right)^{1+q}}{2 b d(3+p+q)}$
$+\frac{1}{2 b^{3} d^{2}(1+p)(2+p+q)(3+p+q)\left(\frac{b\left(d x^{2}+c\right)}{-a d+b c}\right)^{q}}\left(\left(b^{2} c^{2}\left(p^{2}+3 p+2\right)+2 a b c d(1+p)(1+q)+a^{2} d^{2}\left(q^{2}+3 q+2\right)\right)\left(b x^{2}\right.\right.$
$+a)^{1+p}\left(d x^{2}+c\right)^{q}$ hypergeom $\left.\left([-q, 1+p],[2+p],-\frac{d\left(b x^{2}+a\right)}{-a d+b c}\right)\right)$
Result(type 8, 24 leaves):

$$
\int x^{5}\left(b x^{2}+a\right)^{p}\left(d x^{2}+c\right)^{q} \mathrm{~d} x
$$

Test results for the 31 problems in "1.1.2.5 (a+b $\left.x^{\wedge} 2\right)^{\wedge} p\left(c+d x^{\wedge} 2\right)^{\wedge} q\left(e+f x^{\wedge} 2\right)^{\wedge} r . t x t "$
Problem 10: Result more than twice size of optimal antiderivative.

$$
\int\left(b x^{2}+a\right)\left(d x^{2}+c\right)^{3 / 2} \sqrt{f x^{2}+e} \mathrm{~d} x
$$

Optimal(type 4, 566 leaves, 7 steps):
$-\frac{\left(7 a d f\left(-3 c^{2} f^{2}-7 c d e f+2 d^{2} e^{2}\right)-b\left(-6 c^{3} f^{3}+9 c^{2} d e f^{2}-19 c d^{2} e^{2} f+8 d^{3} e^{3}\right)\right) x \sqrt{d x^{2}+c}}{105 d^{2} f^{2} \sqrt{f x^{2}+e}}$

$$
\begin{aligned}
& e^{3 / 2}\left(7 a d f(-9 c f+d e)-b\left(-3 c^{2} f^{2}-9 c d e f+4 d^{2} e^{2}\right)\right) \sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}} \text { EllipticF }\left(\frac{x \sqrt{f}}{\left.\sqrt{e \sqrt{1+\frac{f x^{2}}{e}}}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{d x^{2}+c}}\right. \\
& 105 d f^{5 / 2} \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e} \\
& +\frac{1}{105 d^{2} f^{5} / 2 \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e}}\left(\left(7 a d f\left(-3 c^{2} f^{2}-7 c d e f+2 d^{2} e^{2}\right)-b\left(-6 c^{3} f^{3}+9 c^{2} d e f^{2}-19 c d^{2} e^{2} f\right.\right.\right. \\
& \left.\left.\left.+8 d^{3} e^{3}\right)\right) \sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}} \text { EllipticE }\left(\frac{x \sqrt{f}}{\sqrt{e} \sqrt{1+\frac{f x^{2}}{e}}}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{e} \sqrt{d x^{2}+c}\right)+\frac{(7 a d f-2 b c f+b d e) x\left(d x^{2}+c\right)^{3 / 2} \sqrt{f x^{2}+e}}{35 d f} \\
& +\frac{b x\left(d x^{2}+c\right)^{5 / 2} \sqrt{f x^{2}+e}}{7 d}+\frac{\left(7 a d f(3 c f+d e)-b\left(6 c^{2} f^{2}-6 c d e f+4 d^{2} e^{2}\right)\right) x \sqrt{d x^{2}+c} \sqrt{f x^{2}+e}}{105 d f^{2}} \\
& \text { Result(type 4, } 1331 \text { leaves): } \\
& \frac{1}{105 d\left(d f x^{4}+c f x^{2}+d e x^{2}+c e\right) f^{3} \sqrt{-\frac{d}{c}}}\left(\sqrt { d x ^ { 2 } + c } \sqrt { f x ^ { 2 } + e } \left(42 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c^{2} d e f^{3}\right.\right. \\
& -56 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c d^{2} e^{2} f^{2}-18 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{2} d e^{2} f^{2} \\
& +23 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c d^{2} e^{3} f+21 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c^{2} d e f^{3} \\
& +49 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c d^{2} e^{2} f^{2}+9 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{2} d e^{2} f^{2} \\
& -19 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c d^{2} e^{3} f+18 \sqrt{-\frac{d}{c}} x^{7} b d^{3} e f^{3}+63 \sqrt{-\frac{d}{c}} x^{5} a c d^{2} f^{4}+28 \sqrt{-\frac{d}{c}} x^{5} a d^{3} e f^{3} \\
& +27 \sqrt{-\frac{d}{c}} x^{5} b c^{2} d f^{4}-\sqrt{-\frac{d}{c}} x^{5} b d^{3} e^{2} f^{2}+42 \sqrt{-\frac{d}{c}} x^{3} a c^{2} d f^{4}+7 \sqrt{-\frac{d}{c}} x^{3} a d^{3} e^{2} f^{2}-4 \sqrt{-\frac{d}{c}} x^{3} b d^{3} e^{3} f+3 \sqrt{-\frac{d}{c}} x b c^{3} e f^{3} \\
& -8 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b d^{3} e^{4}+8 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b d^{3} e^{4}+39 \sqrt{-\frac{d}{c}} x^{7} b c d^{2} f^{4} \\
& +15 \sqrt{-\frac{d}{c}} x^{9} b d^{3} f^{4}+21 \sqrt{-\frac{d}{c}} x^{7} a d^{3} f^{4}+3 \sqrt{-\frac{d}{c}} x^{3} b c^{3} f^{4}+14 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a d^{3} e^{3} f \\
& +3 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{3} e f^{3}-14 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a d^{3} e^{3} f
\end{aligned}
$$

$$
\begin{aligned}
& -6 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{3} e f^{3}+51 \sqrt{-\frac{d}{c}} x^{5} b c d^{2} e f^{3}+70 \sqrt{-\frac{d}{c}} x^{3} a c d^{2} e f^{3}+36 \sqrt{-\frac{d}{c}} x^{3} b c^{2} d e f^{3} \\
& \left.\left.+8 \sqrt{-\frac{d}{c}} x^{3} b c d^{2} e^{2} f^{2}+42 \sqrt{-\frac{d}{c}} x a c^{2} d e f^{3}+7 \sqrt{-\frac{d}{c}} x a c d^{2} e^{2} f^{2}+9 \sqrt{-\frac{d}{c}} x b c^{2} d e^{2} f^{2}-4 \sqrt{-\frac{d}{c}} x b c d^{2} e^{3} f\right)\right)
\end{aligned}
$$

Problem 11: Result more than twice size of optimal antiderivative.

$$
\int\left(b x^{2}+a\right) \sqrt{d x^{2}+c}\left(f x^{2}+e\right)^{3 / 2} \mathrm{~d} x
$$

Optimal(type 4, 565 leaves, 7 steps):
$\frac{b x\left(d x^{2}+c\right)^{3 / 2}\left(f x^{2}+e\right)^{3 / 2}}{7 d}+\frac{\left(7 a d f\left(-2 c^{2} f^{2}+7 c d e f+3 d^{2} e^{2}\right)-b\left(-8 c^{3} f^{3}+19 c^{2} d e f^{2}-9 c d^{2} e^{2} f+6 d^{3} e^{3}\right)\right) x \sqrt{d x^{2}+c}}{105 d^{3} f \sqrt{f x^{2}+e}}$

$$
\begin{aligned}
& \quad \begin{array}{l}
e^{3 / 2}\left(7 a d f(-c f+9 d e)-b\left(-4 c^{2} f^{2}+9 c d e f+3 d^{2} e^{2}\right)\right) \sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}} \text { EllipticF }\left(\frac{x \sqrt{f}}{\sqrt{e} \sqrt{1+\frac{f x^{2}}{e}}}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{d x^{2}+c} \\
-\frac{105 d^{2} f^{3} / 2 \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e}}{105 d^{3} f^{3} / 2 \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e}}\left(\left(7 a d f\left(-2 c^{2} f^{2}+7 c d e f+3 d^{2} e^{2}\right)-b\left(-8 c^{3} f^{3}+19 c^{2} d e f^{2}-9 c d^{2} e^{2} f\right.\right.\right.
\end{array}
\end{aligned}
$$

$$
\left.\left.\left.+6 d^{3} e^{3}\right)\right) \sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}} \text { EllipticE }\left(\frac{x \sqrt{f}}{\sqrt{e} \sqrt{1+\frac{f x^{2}}{e}}}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{e} \sqrt{d x^{2}+c}\right)+\frac{(7 a d f-4 b c f+3 b d e) x\left(d x^{2}+c\right)^{3 / 2} \sqrt{f x^{2}+e}}{35 d^{2}}
$$

$+\frac{\left(14 a d f(-c f+3 d e)+b\left(8 c^{2} f^{2}-15 c d e f+3 d^{2} e^{2}\right)\right) x \sqrt{d x^{2}+c} \sqrt{f x^{2}+e}}{105 d^{2} f}$

## Result(type 4, 1331 leaves):

$$
\begin{aligned}
& \frac{1}{105 f^{2}\left(d f x^{4}+c f x^{2}+d e x^{2}+c e\right) d^{2} \sqrt{-\frac{d}{c}}}\left(\sqrt { d x ^ { 2 } + c } \sqrt { f x ^ { 2 } + e } \left(7 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c^{2} d e f^{3}\right.\right. \\
& \quad+14 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \operatorname{EllipticF}\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c d^{2} e^{2} f^{2}+10 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \operatorname{EllipticF}\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{2} d e^{2} f^{2} \\
& \quad-12 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \operatorname{EllipticF}\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c d^{2} e^{3} f-14 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \\
& \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c^{2} d e f^{3}
\end{aligned}
$$

$$
\begin{aligned}
& +49 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c d^{2} e^{2} f^{2}-19 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{2} d e^{2} f^{2} \\
& +9 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c d^{2} e^{3} f+39 \sqrt{-\frac{d}{c}} x^{7} b d^{3} e f^{3}+28 \sqrt{-\frac{d}{c}} x^{5} a c d^{2} f^{4}+63 \sqrt{-\frac{d}{c}} x^{5} a d^{3} e f^{3} \\
& -\sqrt{-\frac{d}{c}} x^{5} b c^{2} d f^{4}+27 \sqrt{-\frac{d}{c}} x^{5} b d^{3} e^{2} f^{2}+7 \sqrt{-\frac{d}{c}} x^{3} a c^{2} d f^{4}+42 \sqrt{-\frac{d}{c}} x^{3} a d^{3} e^{2} f^{2}+3 \sqrt{-\frac{d}{c}} x^{3} b d^{3} e^{3} f-4 \sqrt{-\frac{d}{c}} x b c^{3} e f^{3} \\
& +6 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b d^{3} e^{4}-6 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b d^{3} e^{4}+18 \sqrt{-\frac{d}{c}} x^{7} b c d^{2} f^{4} \\
& +15 \sqrt{-\frac{d}{c}} x^{9} b d^{3} f^{4}+21 \sqrt{-\frac{d}{c}} x^{7} a d^{3} f^{4}-4 \sqrt{-\frac{d}{c}} x^{3} b c^{3} f^{4}-21 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a d^{3} e^{3} f \\
& -4 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{3} e f^{3}+21 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a d^{3} e^{3} f \\
& +8 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{3} e f^{3}+51 \sqrt{-\frac{d}{c} x^{5} b c d^{2} e f^{3}+70 \sqrt{-\frac{d}{c}} x^{3} a c d^{2} e f^{3}+8 \sqrt{-\frac{d}{c}} x^{3} b c^{2} d e f^{3}} \\
& \left.\left.+36 \sqrt{-\frac{d}{c}} x^{3} b c d^{2} e^{2} f^{2}+7 \sqrt{-\frac{d}{c}} x a c^{2} d e f^{3}+42 \sqrt{-\frac{d}{c}} x a c d^{2} e^{2} f^{2}+9 \sqrt{-\frac{d}{c}} x b c^{2} d e^{2} f^{2}+3 \sqrt{-\frac{d}{c}} x b c d^{2} e^{3} f\right)\right)
\end{aligned}
$$

Problem 12: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(b x^{2}+a\right)\left(d x^{2}+c\right)^{5 / 2}}{\sqrt{f x^{2}+e}} \mathrm{~d} x
$$

Optimal(type 4, 573 leaves, 7 steps):
$\underline{\left(7 a d f\left(23 c^{2} f^{2}-23 c d e f+8 d^{2} e^{2}\right)-b\left(-15 c^{3} f^{3}+103 c^{2} d e f^{2}-128 c d^{2} e^{2} f+48 d^{3} e^{3}\right)\right) x \sqrt{d x^{2}+c}}$

$$
105 d f^{3} \sqrt{f x^{2}+e}
$$

$$
\begin{aligned}
& -\frac{1}{105 d f^{7} / 2 \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e}}\left(\begin{array} { l } 
{ ( 7 a d f ( 2 3 c ^ { 2 } f ^ { 2 } - 2 3 c d e f + 8 d ^ { 2 } e ^ { 2 } ) - b ( - 1 5 c ^ { 3 } f ^ { 3 } + 1 0 3 c ^ { 2 } d e f ^ { 2 } - 1 2 8 c d ^ { 2 } e ^ { 2 } f } \\
{ + 4 8 d ^ { 3 } e ^ { 3 } ) ) \sqrt { \frac { 1 } { 1 + \frac { f x ^ { 2 } } { e } } } \sqrt { 1 + \frac { f x ^ { 2 } } { e } } } \\
{ + \frac { 1 } { \sqrt { e } \sqrt { 1 + \frac { f x ^ { 2 } } { e } } } , \sqrt { 1 - \frac { d e } { c f } } ) \sqrt { e } \sqrt { d x ^ { 2 } + c } ) } \\
{ } \\
{ + \frac { x \sqrt { f } } { 1 0 5 f ^ { 7 / 2 } } \sqrt { \frac { e ( d x ^ { 2 } + c ) } { c ( f x ^ { 2 } + e ) } } \sqrt { f x ^ { 2 } + e } }
\end{array} \left(7 a f\left(15 c^{2} f^{2}-11 c d e f+4 d^{2} e^{2}\right)-b e\left(45 c^{2} f^{2}-61 c d e f\right.\right.\right.
\end{aligned}
$$

$\left.\left.+24 d^{2} e^{2}\right)\right) \sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}}$ EllipticF $\left.\left(\frac{x \sqrt{f}}{\sqrt{e} \sqrt{1+\frac{f x^{2}}{e}}}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{e} \sqrt{d x^{2}+c}\right)$
$-\frac{(-7 a d f-5 b c f+6 b d e) x\left(d x^{2}+c\right)^{3 / 2} \sqrt{f x^{2}+e}}{35 f^{2}}+\frac{b x\left(d x^{2}+c\right)^{5 / 2} \sqrt{f x^{2}+e}}{7 f}$
$-\frac{\left(28 a d f(-2 c f+d e)-b\left(15 c^{2} f^{2}-43 c d e f+24 d^{2} e^{2}\right)\right) x \sqrt{d x^{2}+c} \sqrt{f x^{2}+e}}{105 f^{3}}$
Result(type 4, 1385 leaves):
$\frac{1}{105 f^{4}\left(d f x^{4}+c f x^{2}+d e x^{2}+c e\right) \sqrt{-\frac{d}{c}}}\left(\sqrt{d x^{2}+c} \sqrt{f x^{2}+e}\left(-238 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}\right.\right.$ EllipticF $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c^{2} d e f^{3}$
$+189 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}$ EllipticF $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c d^{2} e^{2} f^{2}+164 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}$ EllipticF $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{2} d e^{2} f^{2}$
$-152 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}$ EllipticF $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c d^{2} e^{3} f+161 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}$ EllipticE $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c^{2} d e f^{3}$
$-161 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}$ EllipticE $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c d^{2} e^{2} f^{2}-103 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}$ EllipticE $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{2} d e^{2} f^{2}$
$+128 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}$ EllipticE $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c d^{2} e^{3} f+105 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}$ EllipticF $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c^{3} f^{4}$
$-3 \sqrt{-\frac{d}{c}} x^{7} b d^{3} e f^{3}+98 \sqrt{-\frac{d}{c}} x^{5} a c d^{2} f^{4}-7 \sqrt{-\frac{d}{c}} x^{5} a d^{3} e f^{3}+90 \sqrt{-\frac{d}{c}} x^{5} b c^{2} d f^{4}+6 \sqrt{-\frac{d}{c}} x^{5} b d^{3} e^{2} f^{2}+77 \sqrt{-\frac{d}{c}} x^{3} a c^{2} d f^{4}$
$-28 \sqrt{-\frac{d}{c}} x^{3} a d^{3} e^{2} f^{2}+24 \sqrt{-\frac{d}{c}} x^{3} b d^{3} e^{3} f+45 \sqrt{-\frac{d}{c}} x b c^{3} e f^{3}+48 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}$ EllipticF $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b d^{3} e^{4}$
$-48 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}$ EllipticE $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b d^{3} e^{4}+60 \sqrt{-\frac{d}{c}} x^{7} b c d^{2} f^{4}+15 \sqrt{-\frac{d}{c}} x^{9} b d^{3} f^{4}+21 \sqrt{-\frac{d}{c}} x^{7} a d^{3} f^{4}+45 \sqrt{-\frac{d}{c}} x^{3} b c^{3} f^{4}$
$-56 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}$ EllipticF $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a d^{3} e^{3} f-60 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}$ EllipticF $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{3} e f^{3}$
$+56 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}$ EllipticE $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a d^{3} e^{3} f+15 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}$ EllipticE $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{3} e f^{3}$
$-19 \sqrt{-\frac{d}{c}} x^{5} b c d^{2} e f^{3}+70 \sqrt{-\frac{d}{c}} x^{3} a c d^{2} e f^{3}+29 \sqrt{-\frac{d}{c}} x^{3} b c^{2} d e f^{3}-55 \sqrt{-\frac{d}{c}} x^{3} b c d^{2} e^{2} f^{2}+77 \sqrt{-\frac{d}{c}} x a c^{2} d e f^{3}-28 \sqrt{-\frac{d}{c}} x a c d^{2} e^{2} f^{2}$
$\left.\left.-61 \sqrt{-\frac{d}{c}} x b c^{2} d e^{2} f^{2}+24 \sqrt{-\frac{d}{c}} x b c d^{2} e^{3} f\right)\right)$

Problem 14: Result more than twice size of optimal antiderivative.

$$
\int \frac{b x^{2}+a}{\left(d x^{2}+c\right)^{7 / 2} \sqrt{f x^{2}+e}} \mathrm{~d} x
$$

Optimal(type 4, 435 leaves, 5 steps):

$$
\begin{aligned}
& \left(b c e(-9 c f+d e)+a\left(15 c^{2} f^{2}-11 c d e f+4 d^{2} e^{2}\right)\right) \sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}} \operatorname{EllipticF}\left(\frac{x \sqrt{f}}{\sqrt{e} \sqrt{1+\frac{f x^{2}}{e}}}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{e} \sqrt{f} \sqrt{d x^{2}+c} \\
& 15 c^{3}(-c f+d e)^{3} \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e} \\
& -\frac{(-a d+b c) x \sqrt{f x^{2}+e}}{5 c(-c f+d e)\left(d x^{2}+c\right)^{5 / 2}}+\frac{(4 a d(-2 c f+d e)+b c(3 c f+d e)) x \sqrt{f x^{2}+e}}{15 c^{2}(-c f+d e)^{2}\left(d x^{2}+c\right)^{3 / 2}} \\
& +\frac{1}{15 c^{5 / 2}(-c f+d e)^{3} \sqrt{d} \sqrt{d x^{2}+c} \sqrt{\frac{c\left(f x^{2}+e\right)}{e\left(d x^{2}+c\right)}}}\left(\left(b c\left(-3 c^{2} f^{2}-7 c d e f+2 d^{2} e^{2}\right)+a d\left(23 c^{2} f^{2}-23 c d e f\right.\right.\right. \\
& \left.\left.\left.+8 d^{2} e^{2}\right)\right) \sqrt{\frac{1}{1+\frac{d x^{2}}{c}}} \sqrt{1+\frac{d x^{2}}{c}} \text { EllipticE }\left(\frac{x \sqrt{d}}{\sqrt{c} \sqrt{1+\frac{d x^{2}}{c}}}, \sqrt{1-\frac{c f}{d e}}\right) \sqrt{f x^{2}+e}\right)
\end{aligned}
$$

Result(type ?, 3038 leaves): Display of huge result suppressed!
Problem 16: Result more than twice size of optimal antiderivative.

$$
\int \frac{b x^{2}+a}{\left(d x^{2}+c\right)^{5 / 2}\left(f x^{2}+e\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 409 leaves, 5 steps):

$$
\begin{aligned}
& \frac{(-a d+b c) x}{3 c(-c f+d e)\left(d x^{2}+c\right)^{3 / 2} \sqrt{f x^{2}+e}}+\frac{(2 a d(-3 c f+d e)+b c(3 c f+d e)) x}{3 c^{2}(-c f+d e)^{2} \sqrt{d x^{2}+c} \sqrt{f x^{2}+e}} \\
& +\frac{\left(b c e(7 c f+d e)+a\left(-3 c^{2} f^{2}-7 c d e f+2 d^{2} e^{2}\right)\right) \sqrt{\frac{1}{1+\frac{f x^{2}}{e}} \sqrt{1+\frac{f x^{2}}{e}}} \text { EllipticE }\left(\frac{x \sqrt{f}}{\left.\sqrt{e} \sqrt{1+\frac{f x^{2}}{e}}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{f} \sqrt{d x^{2}+c}}\right.}{3 c^{2}(-c f+d e)^{3} \sqrt{e} \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e}}
\end{aligned}
$$

$$
-\frac{(a d(-9 c f+d e)+b c(3 c f+5 d e)) \sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}} \operatorname{EllipticF}\left(\frac{x \sqrt{f}}{\sqrt{e} \sqrt{1+\frac{f x^{2}}{e}}}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{e} \sqrt{f} \sqrt{d x^{2}+c}}{3 c^{2}(-c f+d e)^{3} \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e}}
$$

Result(type 4, 1741 leaves):

$$
\begin{aligned}
& -\frac{1}{3 \sqrt{f x^{2}+e}(c f-d e)^{3} c^{2} \sqrt{-\frac{d}{c}} e\left(d x^{2}+c\right)^{3 / 2}}\left(2 x^{5} a d^{4} e^{2} f \sqrt{-\frac{d}{c}}-6 x^{3} a c^{3} d f^{3} \sqrt{-\frac{d}{c}}+x^{3} b c d^{3} e^{3} \sqrt{-\frac{d}{c}}+3 x a c d^{3} e^{3} \sqrt{-\frac{d}{c}}+3 x b c^{4} e f^{2} \sqrt{-\frac{d}{c}}\right. \\
& -3 x a c^{4} f^{3} \sqrt{-\frac{d}{c}}+6 \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c^{3} d e f^{2} \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}-8 \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x,\right. \\
& \left.\sqrt{\frac{c f}{d e}}\right) a c^{2} d^{2} e^{2} f \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}-7 \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{3} d e^{2} f \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}+2 \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x\right. \text {, } \\
& \left.\sqrt{\frac{c f}{d e}}\right) b c^{3} d e^{2} f \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}+3 \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c^{3} d e f^{2} \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}+\text { EllipticF }\left(\sqrt{-\frac{d}{c}} x,\right. \\
& \left.\sqrt{\frac{c f}{d e}}\right) x^{2} b c d^{3} e^{3} \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}-\operatorname{EllipticE}\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) x^{2} b c d^{3} e^{3} \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}-3 x^{5} a c^{2} d^{2} f^{3} \sqrt{-\frac{d}{c}} \\
& -7 x^{5} a c d^{3} e f^{2} \sqrt{-\frac{d}{c}}+7 x^{5} b c^{2} d^{2} e f^{2} \sqrt{-\frac{d}{c}}+x^{5} b c d^{3} e^{2} f \sqrt{-\frac{d}{c}}-8 x^{3} a c^{2} d^{2} e f^{2} \sqrt{-\frac{d}{c}}-4 x^{3} a c d^{3} e^{2} f \sqrt{-\frac{d}{c}}+11 x^{3} b c^{3} d e f^{2} \sqrt{-\frac{d}{c}} \\
& +4 x^{3} b c^{2} d^{2} e^{2} f \sqrt{-\frac{d}{c}}-8 x a c^{2} d^{2} e^{2} f \sqrt{-\frac{d}{c}}+5 x b c^{3} d e^{2} f \sqrt{-\frac{d}{c}}+2 \operatorname{EllipticF}\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) x^{2} a d^{4} e^{3} \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \\
& \text { - 2 EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) x^{2} a d^{4} e^{3} \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}+2 \operatorname{EllipticF}\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c d^{3} e^{3} \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \\
& \text { - 3 EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{4} e f^{2} \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}+\operatorname{EllipticF}\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{2} d^{2} e^{3} \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \\
& \text { - } 2 \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c d^{3} e^{3} \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}-\operatorname{EllipticE}\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) b c^{2} d^{2} e^{3} \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}+2 x^{3} a d^{4} e^{3} \sqrt{-\frac{d}{c}} \\
& +3 \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) x^{2} a c^{2} d^{2} e f^{2} \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}+7 \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) x^{2} a c d^{3} e^{2} f \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \\
& -7 \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) x^{2} b c^{2} d^{2} e^{2} f \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}+6 \operatorname{EllipticF}\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) x^{2} a c^{2} d^{2} e f^{2} \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \\
& -8 \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) x^{2} a c d^{3} e^{2} f \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}-3 \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) x^{2} b c^{3} d e f^{2} \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}
\end{aligned}
$$

+2 EllipticF $\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) x^{2} b c^{2} d^{2} e^{2} f \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}+7$ EllipticE $\left.\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a c^{2} d^{2} e^{2} f \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}\right)$

Problem 18: Unable to integrate problem.

$$
\int \frac{2 c x^{2}-\sqrt{-4 a c+b^{2}}+b}{\sqrt{1+\frac{2 c x^{2}}{b-\sqrt{-4 a c+b^{2}}}} \sqrt{1+\frac{2 c x^{2}}{b+\sqrt{-4 a c+b^{2}}}}} \mathrm{~d} x
$$

Optimal(type 4, 499 leaves, 5 steps):
$\frac{x\left(b-\sqrt{-4 a c+b^{2}}\right) \sqrt{1+\frac{2 c x^{2}}{b-\sqrt{-4 a c+b^{2}}}}}{\sqrt{1+\frac{2 c x^{2}}{b+\sqrt{-4 a c+b^{2}}}}}$

$\left.\left.\sqrt{-\frac{2 \sqrt{-4 a c+b^{2}}}{b-\sqrt{-4 a c+b^{2}}}}\right)\left(b-\sqrt{-4 a c+b^{2}}\right) \sqrt{1+\frac{2 c x^{2}}{b-\sqrt{-4 a c+b^{2}}}} \sqrt{b+\sqrt{-4 a c+b^{2}}} \sqrt{2}\right)$

$\left.\left.\sqrt{-\frac{2 \sqrt{-4 a c+b^{2}}}{b-\sqrt{-4 a c+b^{2}}}}\right)\left(b-\sqrt{-4 a c+b^{2}}\right) \sqrt{1+\frac{2 c x^{2}}{b-\sqrt{-4 a c+b^{2}}}} \sqrt{b+\sqrt{-4 a c+b^{2}}} \sqrt{2}\right)$

Result(type 8, 73 leaves):

$$
\int \frac{2 c x^{2}-\sqrt{-4 a c+b^{2}}+b}{\sqrt{1+\frac{2 c x^{2}}{b-\sqrt{-4 a c+b^{2}}}} \sqrt{1+\frac{2 c x^{2}}{b+\sqrt{-4 a c+b^{2}}}}} \mathrm{~d} x
$$

Problem 19: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(b x^{2}+a\right)^{2}}{\left(d x^{2}+c\right) \sqrt{f x^{2}+e}} d x
$$

Optimal(type 3, 138 leaves, 9 steps):

$$
-\frac{b(-2 a f+b e) \operatorname{arctanh}\left(\frac{x \sqrt{f}}{\sqrt{f x^{2}+e}}\right)}{2 d f^{3 / 2}}-\frac{b(-a d+b c) \operatorname{arctanh}\left(\frac{x \sqrt{f}}{\sqrt{f x^{2}+e}}\right)}{d^{2} \sqrt{f}}+\frac{(-a d+b c)^{2} \arctan \left(\frac{x \sqrt{-c f+d e}}{\sqrt{c} \sqrt{f x^{2}+e}}\right)}{d^{2} \sqrt{c} \sqrt{-c f+d e}}+\frac{b^{2} x \sqrt{f x^{2}+e}}{2 d f}
$$

Result(type 3, 1051 leaves):

$$
\frac{b^{2} x \sqrt{f x^{2}+e}}{2 d f}-\frac{b^{2} e \ln \left(\sqrt{f} x+\sqrt{f x^{2}+e}\right)}{2 d f^{3 / 2}}+\frac{2 b a \ln \left(\sqrt{f} x+\sqrt{f x^{2}+e}\right)}{d \sqrt{f}}-\frac{b^{2} c \ln \left(\sqrt{f} x+\sqrt{f x^{2}+e}\right)}{d^{2} \sqrt{f}}
$$

$$
\ln \left(\frac{-\frac{2(c f-d e)}{d}-\frac{2 f \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{-\frac{c f-d e}{d}} \sqrt{\left(x+\frac{\sqrt{-c d}}{d}\right)^{2} f-\frac{2 f \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}-\frac{c f-d e}{d}}}{x+\frac{\sqrt{-c d}}{d}}\right) a^{2}
$$

$$
-\frac{\ln \left(\frac{-\frac{2(c f-d e)}{d}-\frac{2 f \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{-\frac{c f-d e}{d}} \sqrt{\left(x+\frac{\sqrt{-c d}}{d}\right)^{2} f-\frac{2 f \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}-\frac{c f-d e}{d}}}{x+\frac{\sqrt{-c d}}{d}}\right) a c b}{d \sqrt{-c d} \sqrt{-\frac{c f-d e}{d}}}
$$

$$
\begin{aligned}
& \ln \left(\frac{-\frac{2(c f-d e)}{d}-\frac{2 f \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{-\frac{c f-d e}{d}} \sqrt{\left(x+\frac{\sqrt{-c d}}{d}\right)^{2} f-\frac{2 f \sqrt{-c d}\left(x+\frac{\sqrt{-c d}}{d}\right)}{d}-\frac{c f-d e}{d}}}{x+\frac{\sqrt{-c d}}{d}}\right) b^{2} c^{2} \\
& 2 d^{2} \sqrt{-c d} \sqrt{-\frac{c f-d e}{d}} \\
& \left.-\frac{\ln \left(\frac{-\frac{2(c f-d e)}{d}+\frac{2 f \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{-\frac{c f-d e}{d}} \sqrt{\left(x-\frac{\sqrt{-c d}}{d}\right)^{2} f+\frac{2 f \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}-\frac{c f-d e}{d}}}{x-\frac{\sqrt{-c d}}{d}}\right)}{2 \sqrt{-c d} \sqrt{-\frac{c f-d e}{d}}}\right) \\
& \ln \left(\frac{-\frac{2(c f-d e)}{d}+\frac{2 f \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{-\frac{c f-d e}{d}} \sqrt{\left(x-\frac{\sqrt{-c d}}{d}\right)^{2} f+\frac{2 f \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}-\frac{c f-d e}{d}}}{x-\frac{\sqrt{-c d}}{d}}\right) a c b \\
& \ln \left(\frac{\left.-\frac{2(c f-d e)}{d}+\frac{2 f \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}+2 \sqrt{-\frac{c f-d e}{d}} \sqrt{\left(x-\frac{\sqrt{-c d}}{d}\right)^{2} f+\frac{2 f \sqrt{-c d}\left(x-\frac{\sqrt{-c d}}{d}\right)}{d}-\frac{c f-d e}{d}}\right)}{x-\frac{\sqrt{-c d}}{d}}\right) b^{2} c^{2} \\
& 2 d^{2} \sqrt{-c d} \sqrt{-\frac{c f-d e}{d}}
\end{aligned}
$$

Problem 20: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(d x^{2}+c\right)^{5 / 2} \sqrt{f x^{2}+e}}{b x^{2}+a} d x
$$

Optimal(type 4, 649 leaves, 14 steps):
$\frac{d^{2} x\left(f x^{2}+e\right)^{3 / 2} \sqrt{d x^{2}+c}}{5 b f}+\frac{d\left(7 c e-\frac{2 d e^{2}}{f}+\frac{3 c^{2} f}{d}\right) x \sqrt{d x^{2}+c}}{15 b \sqrt{f x^{2}+e}}+\frac{(-a d+b c)(-3 a d f+4 b c f+b d e) x \sqrt{d x^{2}+c}}{3 b^{3} \sqrt{f x^{2}+e}}$

$$
\begin{aligned}
& +\frac{d e^{3 / 2}\left(-40 a b c d f+15 a^{2} d^{2} f+b^{2} c(34 c f-d e)\right) \sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}} \operatorname{EllipticF}\left(\frac{x \sqrt{f}}{\sqrt{e} \sqrt{1+\frac{f x^{2}}{e}}}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{d x^{2}+c}}{15 b^{3} c f^{3 / 2} \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e}} \\
& -\frac{1}{15 b^{3} f^{3} / 2 \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e}}\left(\left(15 a^{2} d^{2} f^{2}-5 a b d f(7 c f+d e)+b^{2}\left(23 c^{2} f^{2}+12 c d e f\right.\right.\right. \\
& \left.\left.\left.-2 d^{2} e^{2}\right)\right) \sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}} \operatorname{EllipticE}\left(\frac{x \sqrt{f}}{\sqrt{e} \sqrt{1+\frac{f x^{2}}{e}}}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{e} \sqrt{d x^{2}+c}\right) \\
& +\frac{(-a d+b c)^{3} e^{3 / 2}}{\sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}} \operatorname{EllipticPi}\left(\frac{x \sqrt{f}}{\sqrt{e} \sqrt{1+\frac{f x^{2}}{e}}}, 1-\frac{b e}{a f}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{d x^{2}+c}} \\
& +\frac{a b^{3} c \sqrt{f} \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)} \sqrt{f x^{2}+e}}}{}=1
\end{aligned}
$$

$$
-\frac{2 d(-3 c f+d e) x \sqrt{d x^{2}+c} \sqrt{f x^{2}+e}}{15 b f}
$$

Result(type 4, 1890 leaves):
$-\frac{1}{15\left(d f x^{4}+c f x^{2}+d e x^{2}+c e\right) b^{4} f^{2} \sqrt{-\frac{d}{c}} a}\left(\sqrt{d x^{2}+c} \sqrt{f x^{2}+e}\left(-3 \sqrt{-\frac{d}{c}} x^{7} a b^{3} d^{3} f^{3}+15 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}\right.\right.$ EllipticF $\left(\sqrt{-\frac{d}{c}} x\right.$,

$$
\begin{aligned}
& \left.\sqrt{\frac{c f}{d e}}\right) a^{4} d^{3} f^{3}-15 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi }\left(\sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) a^{4} d^{3} f^{3}-5 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x\right. \\
& \left.\sqrt{\frac{c f}{d e}}\right) a^{2} b^{2} d^{3} e^{2} f-15 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \operatorname{EllipticE}\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a^{3} b d^{3} e f^{2}+5 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \operatorname{EllipticE}\left(\sqrt{-\frac{d}{c}} x\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left.\sqrt{\frac{c f}{d e}}\right) a^{2} b^{2} d^{3} e^{2} f+45 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi }\left(\sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) a^{3} b c d^{2} f^{3}+15 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi } \sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \\
& \left.\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) a^{3} b d^{3} e f^{2}-45 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi }\left(\sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) a^{2} b^{2} c^{2} d f^{3}-15 \sqrt{-\frac{d}{c}} x^{3} a b^{3} c d^{2} e f^{2}+5 \sqrt{-\frac{d}{c}} x a^{2} b^{2} c d^{2} e f^{2} \\
& -11 \sqrt{-\frac{d}{c}} x a b^{3} c^{2} d e f^{2}-\sqrt{-\frac{d}{c}} x a b^{3} c d^{2} e^{2} f-45 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a^{3} b c d^{2} f^{3} \\
& +45 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a^{2} b^{2} c^{2} d f^{3}+35 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a^{2} b^{2} c d^{2} e f^{2} \\
& -23 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a b^{3} c^{2} d e f^{2}-12 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a b^{3} c d^{2} e^{2} f \\
& \left.-45 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi } \sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) a^{2} b^{2} c d^{2} e f^{2}+45 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi } \sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \\
& \left.\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) a b^{3} c^{2} d e f^{2}+5 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \operatorname{EllipticF}\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a^{2} b^{2} c d^{2} e f^{2}-11 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x,\right. \\
& \left.\sqrt{\frac{c f}{d e}}\right) a b^{3} c^{2} d e f^{2}+13 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a b^{3} c d^{2} e^{2} f+5 \sqrt{-\frac{d}{c}} x^{5} a^{2} b^{2} d^{3} f^{3} \\
& -15 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a b^{3} c^{3} f^{3}-2 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a b^{3} d^{3} e^{3}
\end{aligned}
$$

$$
\begin{aligned}
& \left.+2 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a b^{3} d^{3} e^{3}+15 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi } \sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) a b^{3} c^{3} f^{3} \\
& -15 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi }\left(\sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) b^{4} c^{3} e f^{2}-14 \sqrt{-\frac{d}{c}} x^{5} a b^{3} c d^{2} f^{3}-4 \sqrt{-\frac{d}{c}} x^{5} a b^{3} d^{3} e f^{2}+5 \sqrt{-\frac{d}{c}} x^{3} a^{2} b^{2} c d^{2} f^{3} \\
& \left.+5 \sqrt{-\frac{d}{c}} x^{3} a^{2} b^{2} d^{3} e f^{2}-11 \sqrt{-\frac{d}{c}} x^{3} a b^{3} c^{2} d f^{3}-\sqrt{-\frac{d}{c}} x^{3} a b^{3} d^{3} e^{2} f\right)
\end{aligned}
$$

Problem 21: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(d x^{2}+c\right)^{3 / 2} \sqrt{f x^{2}+e}}{b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 4, 459 leaves, 7 steps):

$$
\begin{aligned}
& \frac{(-3 a d f+4 b c f+b d e) x \sqrt{d x^{2}+c}}{3 b^{2} \sqrt{f x^{2}+e}}+\frac{d(-3 a d+5 b c) e^{3 / 2} \sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}} \text { EllipticF }\left(\frac{x \sqrt{f}}{\left.\sqrt{e} \sqrt{1+\frac{f x^{2}}{e}}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{d x^{2}+c}}\right)}{3 b^{2} c \sqrt{f} \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e}} \\
& +\frac{(-a d+b c)^{2} e^{3 / 2} \sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}} \operatorname{EllipticPi}\left(\frac{x \sqrt{f}}{\sqrt{e} \sqrt{1+\frac{f x^{2}}{e}}}, 1-\frac{b e}{a f}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{d x^{2}+c}}{a b^{2} c \sqrt{f} \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e}} \\
& \frac{(-3 a d f+4 b c f+b d e) \sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}} \text { EllipticE }\left(\frac{x \sqrt{f}}{\sqrt{e} \sqrt{1+\frac{f x^{2}}{e}}}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{e} \sqrt{d x^{2}+c}}{3 b^{2} \sqrt{f} \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e}}+\frac{d x \sqrt{d x^{2}+c} \sqrt{f x^{2}+e}}{3 b}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{3\left(d f x^{4}+c f x^{2}+d e x^{2}+c e\right) b^{3} \sqrt{-\frac{d}{c}} f a}\left(\sqrt { d x ^ { 2 } + c } \sqrt { f x ^ { 2 } + e } \left(\sqrt{-\frac{d}{c}} x^{5} a b^{2} d^{2} f^{2}+\sqrt{-\frac{d}{c}} x^{3} a b^{2} c d f^{2}+\sqrt{-\frac{d}{c}} x^{3} a b^{2} d^{2} e f\right.\right. \\
& +3 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a^{3} d^{2} f^{2}-6 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a^{2} b c d f^{2} \\
& +3 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a b^{2} c^{2} f^{2}+\sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a b^{2} c d e f \\
& -\sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a b^{2} d^{2} e^{2}-3 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a^{2} b d^{2} e f \\
& +4 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a b^{2} c d e f+\sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a b^{2} d^{2} e^{2} \\
& \left.\left.\left.-3 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi } \sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) a^{3} d^{2} f^{2}+6 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi } \sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) \text { ar }\right) \\
& +3 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi }\left(\sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) a^{2} b d^{2} e f-3 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \operatorname{EllipticPi}\left(\sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) a b^{2} c^{2} f^{2} \\
& \left.-6 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi }\left(\sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) a b^{2} c d e f+3 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \operatorname{EllipticPi} \sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) b^{3} c^{2} e f \\
& \left.+\sqrt{-\frac{d}{c}} x a b^{2} c d e f\right)
\end{aligned}
$$

Problem 23: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{f x^{2}+e}}{\left(b x^{2}+a\right)\left(d x^{2}+c\right)^{7 / 2}} \mathrm{~d} x
$$

Optimal(type 4, 714 leaves, 9 steps):

$$
\begin{aligned}
& a c(-a d+b c)^{3} \sqrt{f} \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e} \\
& d e^{3 / 2}(b c(-11 c f+9 d e)-2 a d(-3 c f+2 d e)) \sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}} \operatorname{EllipticF}\left(\frac{x \sqrt{f}}{\sqrt{e} \sqrt{1+\frac{f x^{2}}{e}}}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{f} \sqrt{d x^{2}+c} \\
& 15 c^{3}(-a d+b c)^{2}(-c f+d e)^{2} \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e} \\
& -\frac{d x \sqrt{f x^{2}+e}}{5 c(-a d+b c)\left(d x^{2}+c\right)^{5 / 2}}-\frac{d(b c(-8 c f+9 d e)-a d(-3 c f+4 d e)) x \sqrt{f x^{2}+e}}{15 c^{2}(-a d+b c)^{2}(-c f+d e)\left(d x^{2}+c\right)^{3 / 2}} \\
& +\frac{1}{15 c^{5 / 2}(-a d+b c)^{2}(-c f+d e)^{2} \sqrt{d x^{2}+c} \sqrt{\frac{c\left(f x^{2}+e\right)}{e\left(d x^{2}+c\right)}}}\left(\left(a d\left(3 c^{2} f^{2}-13 c d e f+8 d^{2} e^{2}\right)-2 b c\left(4 c^{2} f^{2}-14 c d e f\right.\right.\right. \\
& \left.\left.\left.+9 d^{2} e^{2}\right)\right) \sqrt{\frac{1}{1+\frac{d x^{2}}{c}}} \sqrt{1+\frac{d x^{2}}{c}} \text { EllipticE }\left(\frac{x \sqrt{d}}{\sqrt{c} \sqrt{1+\frac{d x^{2}}{c}}}, \sqrt{1-\frac{c f}{d e}}\right) \sqrt{d} \sqrt{f x^{2}+e}\right) \\
& -\sqrt{b^{2} \sqrt{\frac{1}{1+\frac{d x^{2}}{c}}} \sqrt{1+\frac{d x^{2}}{c}} \text { EllipticE }\left(\frac{x \sqrt{d}}{\sqrt{c} \sqrt{1+\frac{d x^{2}}{c}}}, \sqrt{1-\frac{c f}{d e}}\right) \sqrt{d} \sqrt{f x^{2}+e}} \\
& (-a d+b c)^{3} \sqrt{c} \sqrt{d x^{2}+c} \sqrt{\frac{c\left(f x^{2}+e\right)}{e\left(d x^{2}+c\right)}}
\end{aligned}
$$

Result(type ?, 6244 leaves): Display of huge result suppressed!
Problem 29: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{\left(b x^{2}+a\right)^{2} \sqrt{-d x^{2}+c} \sqrt{f x^{2}+e}} \mathrm{~d} x
$$

Optimal(type 4, 369 leaves, 11 steps):

$$
\begin{aligned}
& \frac{b^{2} x \sqrt{-d x^{2}+c} \sqrt{f x^{2}+e}}{2 a(a d+b c)(-a f+b e)\left(b x^{2}+a\right)}+\frac{b \text { EllipticE }\left(\frac{x \sqrt{d}}{\sqrt{c}}, \sqrt{-\frac{c f}{d e}}\right) \sqrt{c} \sqrt{d} \sqrt{1-\frac{d x^{2}}{c}} \sqrt{f x^{2}+e}}{2 a(a d+b c)(-a f+b e) \sqrt{-d x^{2}+c} \sqrt{1+\frac{f x^{2}}{e}}} \\
& +\frac{\left(b^{2} c e-3 a^{2} d f+a b(-2 c f+2 d e)\right) \operatorname{EllipticPi}\left(\frac{x \sqrt{d}}{\sqrt{c}},-\frac{b c}{a d}, \sqrt{-\frac{c f}{d e}}\right) \sqrt{c} \sqrt{1-\frac{d x^{2}}{c}} \sqrt{1+\frac{f x^{2}}{e}}}{2 a^{2}(a d+b c)(-a f+b e) \sqrt{d} \sqrt{-d x^{2}+c} \sqrt{f x^{2}+e}}
\end{aligned}
$$

$$
-\underline{\text { EllipticF }\left(\frac{x \sqrt{d}}{\sqrt{c}}, \sqrt{-\frac{c f}{d e}}\right) \sqrt{c} \sqrt{d} \sqrt{1-\frac{d x^{2}}{c}} \sqrt{1+\frac{f x^{2}}{e}}}
$$

$$
2 a(a d+b c) \sqrt{-d x^{2}+c} \sqrt{f x^{2}+e}
$$

Result(type 4, 1104 leaves):

$$
\begin{aligned}
& \frac{1}{2 \sqrt{\frac{d}{c}}\left(b x^{2}+a\right) a^{2}(a f-b e)(a d+b c)\left(d f x^{4}-c f x^{2}+d e x^{2}-c e\right)}\left(\sqrt { \frac { d } { c } } x ^ { 5 } a b ^ { 2 } d f + \sqrt { \frac { f x ^ { 2 } + e } { e } } \sqrt { - \frac { d x ^ { 2 } - c } { c } } \text { EllipticF } \left(x \sqrt{\frac{d}{c}},\right.\right. \\
& \left.\sqrt{-\frac{c f}{d e}}\right) x^{2} a^{2} b d f-\sqrt{\frac{f x^{2}+e}{e}} \sqrt{-\frac{d x^{2}-c}{c}} \text { EllipticF }\left(x \sqrt{\frac{d}{c}}, \sqrt{-\frac{c f}{d e}}\right) x^{2} a b^{2} d e+\sqrt{\frac{f x^{2}+e}{e}} \sqrt{-\frac{d x^{2}-c}{c}} \text { EllipticE }\left(x \sqrt{\frac{d}{c}},\right. \\
& \left.\sqrt{-\frac{c f}{d e}}\right) x^{2} a b^{2} d e-3 \sqrt{\frac{f x^{2}+e}{e}} \sqrt{-\frac{d x^{2}-c}{c}} \text { EllipticPi }\left(x \sqrt{\frac{d}{c}},-\frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{\frac{d}{c}}}\right) x^{2} a^{2} b d f-2 \sqrt{\frac{f x^{2}+e}{e}} \sqrt{-\frac{d x^{2}-c}{c}} \text { EllipticPi } x \sqrt{\frac{d}{c}},-\frac{b c}{a d}, \\
& \left.\frac{\sqrt{-\frac{f}{e}}}{\sqrt{\frac{d}{c}}}\right) x^{2} a b^{2} c f+2 \sqrt{\frac{f x^{2}+e}{e}} \sqrt{-\frac{d x^{2}-c}{c}} \text { EllipticPi }\left(x \sqrt{\frac{d}{c}},-\frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{\frac{d}{c}}}\right) x^{2} a b^{2} d e+\sqrt{\frac{f x^{2}+e}{e}} \sqrt{-\frac{d x^{2}-c}{c}} \text { EllipticPi } x \sqrt{\frac{d}{c}},-\frac{b c}{a d}, \\
& \left.\frac{\sqrt{-\frac{f}{e}}}{\sqrt{\frac{d}{c}}}\right) x^{2} b^{3} c e+\sqrt{\frac{d}{c}} x^{3} a b^{2} c f-\sqrt{\frac{d}{c}} x^{3} a b^{2} d e+\sqrt{\frac{f x^{2}+e}{e}} \sqrt{-\frac{d x^{2}-c}{c}} \text { EllipticF }\left(x \sqrt{\frac{d}{c}}, \sqrt{-\frac{c f}{d e}}\right) a^{3} d f
\end{aligned}
$$

$$
-\sqrt{\frac{f x^{2}+e}{e}} \sqrt{-\frac{d x^{2}-c}{c}} \text { EllipticF }\left(x \sqrt{\frac{d}{c}}, \sqrt{-\frac{c f}{d e}}\right) a^{2} b d e+\sqrt{\frac{f x^{2}+e}{e}} \sqrt{-\frac{d x^{2}-c}{c}} \text { EllipticE }\left(x \sqrt{\frac{d}{c}}, \sqrt{-\frac{c f}{d e}}\right) a^{2} b d e
$$

$$
-3 \sqrt{\frac{f x^{2}+e}{e}} \sqrt{-\frac{d x^{2}-c}{c}} \text { EllipticPi }\left(x \sqrt{\frac{d}{c}},-\frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{\frac{d}{c}}}\right) a^{3} d f-2 \sqrt{\frac{f x^{2}+e}{e}} \sqrt{-\frac{d x^{2}-c}{c}} \text { EllipticPi }\left(x \sqrt{\frac{d}{c}},-\frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{\frac{d}{c}}}\right) a^{a^{2} b c f}
$$

$$
\left.+2 \sqrt{\frac{f x^{2}+e}{e}} \sqrt{-\frac{d x^{2}-c}{c}} \text { EllipticPi }\left(x \sqrt{\frac{d}{c}},-\frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{\frac{d}{c}}}\right) a^{2} b d e+\sqrt{\frac{f x^{2}+e}{e}} \sqrt{-\frac{d x^{2}-c}{c}} \text { EllipticPi } x \sqrt{\frac{d}{c}},-\frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{\frac{d}{c}}}\right) a b^{2} c e
$$

$$
\left.\left.+\sqrt{\frac{d}{c}} x a b^{2} c e\right) \sqrt{f x^{2}+e} \sqrt{-d x^{2}+c}\right)
$$

Problem 30: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{\left(b x^{2}+a\right)^{2} \sqrt{d x^{2}+c} \sqrt{f x^{2}+e}} \mathrm{~d} x
$$

Optimal(type 4, 502 leaves, 8 steps):
$-\frac{b f x \sqrt{d x^{2}+c}}{2 a(-a d+b c)(-a f+b e) \sqrt{f x^{2}+e}}+\frac{b \sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}} \operatorname{EllipticE}\left(\frac{x \sqrt{f}}{\left.\sqrt{e \sqrt{1+\frac{f x^{2}}{e}}}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{e} \sqrt{f} \sqrt{d x^{2}+c}}\right)}{2 a(-a d+b c)(-a f+b e) \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e}}$

$$
-\frac{d \sqrt{\frac{1}{1+\frac{f x^{2}}{e}}} \sqrt{1+\frac{f x^{2}}{e}} \text { EllipticF }\left(\frac{x \sqrt{f}}{\sqrt{e} \sqrt{1+\frac{f x^{2}}{e}}}, \sqrt{1-\frac{d e}{c f}}\right) \sqrt{e} \sqrt{f} \sqrt{d x^{2}+c}}{2 c(-a d+b c)(-a f+b e) \sqrt{\frac{e\left(d x^{2}+c\right)}{c\left(f x^{2}+e\right)}} \sqrt{f x^{2}+e}}+\frac{b^{2} x \sqrt{d x^{2}+c} \sqrt{f x^{2}+e}}{2 a(-a d+b c)(-a f+b e)\left(b x^{2}+a\right)}
$$

$$
+\frac{\left(b^{2} c e+3 a^{2} d f-2 a b(c f+d e)\right) \text { EllipticPi }\left(\frac{x \sqrt{d}}{\sqrt{-c}}, \frac{b c}{a d}, \sqrt{\frac{c f}{d e}}\right) \sqrt{-c} \sqrt{1+\frac{d x^{2}}{c}} \sqrt{1+\frac{f x^{2}}{e}}}{2 a^{2}(-a d+b c)(-a f+b e) \sqrt{d} \sqrt{d x^{2}+c} \sqrt{f x^{2}+e}}
$$

Result(type 4, 1077 leaves):

$$
-3 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi }\left(\sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) x^{2} a^{2} b d f+2 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi }\left(\sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) x^{2} a b^{2} c f
$$

$$
+2 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi }\left(\sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) x^{2} a b^{2} d e-\sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi }\left(\sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) x^{2} b^{3} c e
$$

$$
-\sqrt{-\frac{d}{c}} x^{3} a b^{2} c f-\sqrt{-\frac{d}{c}} x^{3} a b^{2} d e+\sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a^{3} d f-\sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x\right.
$$

$$
\left.\sqrt{\frac{c f}{d e}}\right) a^{2} b d e+\sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) a^{2} b d e-3 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi }\left(\sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) a^{3} d f
$$

$$
+2 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi }\left(\sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) a^{2} b c f+2 \sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticPi }\left(\sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) a^{2} b d e
$$

$$
\begin{aligned}
& -\frac{1}{2 \sqrt{-\frac{d}{c}}\left(b x^{2}+a\right) a^{2}(a f-b e)(a d-b c)\left(d f x^{4}+c f x^{2}+d e x^{2}+c e\right)}\left(\left(- \sqrt { - \frac { d } { c } } x ^ { 5 } a b ^ { 2 } d f + \sqrt { \frac { d x ^ { 2 } + c } { c } } \sqrt { \frac { f x ^ { 2 } + e } { e } } \text { EllipticF } \left(\sqrt{-\frac{d}{c}} x,\right.\right.\right. \\
& \left.\sqrt{\frac{c f}{d e}}\right) x^{2} a^{2} b d f-\sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticF }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) x^{2} a b^{2} d e+\sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}} \text { EllipticE }\left(\sqrt{-\frac{d}{c}} x, \sqrt{\frac{c f}{d e}}\right) x^{2} a b^{2} d e
\end{aligned}
$$

$-\sqrt{\frac{d x^{2}+c}{c}} \sqrt{\frac{f x^{2}+e}{e}}$ EllipticPi $\left.\left.\left(\sqrt{-\frac{d}{c}} x, \frac{b c}{a d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) a b^{2} c e-\sqrt{-\frac{d}{c}} x a b^{2} c e\right) \sqrt{f x^{2}+e} \sqrt{d x^{2}+c}\right)$

Problem 31: Unable to integrate problem.

$$
\int \frac{\sqrt{b x^{2}+a}}{\sqrt{d x^{2}+c} \sqrt{f x^{2}+e}} \mathrm{~d} x
$$

Optimal(type 4, 142 leaves, 2 steps):

$$
a \text { EllipticPi }\left(\frac{x \sqrt{-a d+b c}}{\sqrt{c} \sqrt{b x^{2}+a}}, \frac{b c}{-a d+b c}, \sqrt{\frac{c(-a f+b e)}{(-a d+b c) e}}\right) \sqrt{d x^{2}+c} \sqrt{\frac{a\left(f x^{2}+e\right)}{e\left(b x^{2}+a\right)}}
$$

$$
\sqrt{c} \sqrt{-a d+b c} \sqrt{\frac{a\left(d x^{2}+c\right)}{c\left(b x^{2}+a\right)}} \sqrt{f x^{2}+e}
$$

Result(type 8, 30 leaves):

$$
\int \frac{\sqrt{b x^{2}+a}}{\sqrt{d x^{2}+c} \sqrt{f x^{2}+e}} \mathrm{~d} x
$$


Problem 1: Result more than twice size of optimal antiderivative.

$$
\int(e x)^{m}\left(b x^{2}+a\right)^{3}\left(B x^{2}+A\right)\left(d x^{2}+c\right) \mathrm{d} x
$$

Optimal(type 3, 189 leaves, 2 steps):

$$
\begin{aligned}
& \frac{a^{3} A c(e x)^{1+m}}{e(1+m)}+\frac{a^{2}(a A d+3 A b c+a B c)(e x)^{3+m}}{e^{3}(3+m)}+\frac{a(3 A b(a d+b c)+a B(a d+3 b c))(e x)^{5+m}}{e^{5}(5+m)} \\
& \quad+\frac{b(3 a B(a d+b c)+A b(3 a d+b c))(e x)^{7+m}}{e^{7}(7+m)}+\frac{b^{2}(A b d+3 a B d+b B c)(e x)^{9+m}}{e^{9}(9+m)}+\frac{b^{3} B d(e x)^{11+m}}{e^{11}(11+m)}
\end{aligned}
$$

Result(type 3, 1228 leaves):
$\frac{1}{(11+m)(9+m)(7+m)(5+m)(3+m)(1+m)}\left(x\left(B b^{3} d m^{5} x^{10}+25 B b^{3} d m^{4} x^{10}+A b^{3} d m^{5} x^{8}+3 B a b^{2} d m^{5} x^{8}+B b^{3} c m^{5} x^{8}+230 B b^{3} d m^{3} x^{10}\right.\right.$
$+27 A b^{3} d m^{4} x^{8}+81 B a b^{2} d m^{4} x^{8}+27 B b^{3} c m^{4} x^{8}+950 B b^{3} d m^{2} x^{10}+3 A a b^{2} d m^{5} x^{6}+A b^{3} c m^{5} x^{6}+262 A b^{3} d m^{3} x^{8}+3 B a^{2} b d m^{5} x^{6}+3 B a b^{2} c m^{5} x^{6}$
$+786 B a b^{2} d m^{3} x^{8}+262 B b^{3} c m^{3} x^{8}+1689 B b^{3} d m x^{10}+87 A a b^{2} d m^{4} x^{6}+29 A b^{3} c m^{4} x^{6}+1122 A b^{3} d m^{2} x^{8}+87 B a^{2} b d m^{4} x^{6}+87 B a b^{2} c m^{4} x^{6}$
$+3366 B a b^{2} d m^{2} x^{8}+1122 B b^{3} c m^{2} x^{8}+945 B d b^{3} x^{10}+3 A a^{2} b d m^{5} x^{4}+3 A a b^{2} c m^{5} x^{4}+906 A a b^{2} d m^{3} x^{6}+302 A b^{3} c m^{3} x^{6}+2041 A b^{3} d m x^{8}$
$+B a^{3} d m^{5} x^{4}+3 B a^{2} b c m^{5} x^{4}+906 B a^{2} b d m^{3} x^{6}+906 B a b^{2} c m^{3} x^{6}+6123 B a b^{2} d m x^{8}+2041 B b^{3} c m x^{8}+93 A a^{2} b d m^{4} x^{4}+93 A a b^{2} c m^{4} x^{4}$

```
+4098Aab d dm 2}\mp@subsup{x}{}{6}+1366A\mp@subsup{b}{}{3}c\mp@subsup{m}{}{2}\mp@subsup{x}{}{6}+1155A\mp@subsup{b}{}{3}d\mp@subsup{x}{}{8}+31B\mp@subsup{a}{}{3}d\mp@subsup{m}{}{4}\mp@subsup{x}{}{4}+93B\mp@subsup{a}{}{2}bc\mp@subsup{m}{}{4}\mp@subsup{x}{}{4}+4098B\mp@subsup{a}{}{2}bd\mp@subsup{m}{}{2}\mp@subsup{x}{}{6}+4098Ba\mp@subsup{b}{}{2}c\mp@subsup{m}{}{2}\mp@subsup{x}{}{6}+3465Ba\mp@subsup{b}{}{2}d\mp@subsup{x}{}{8
+1155B\mp@subsup{b}{}{3}c\mp@subsup{x}{}{8}+A\mp@subsup{a}{}{3}d\mp@subsup{m}{}{5}\mp@subsup{x}{}{2}+3A\mp@subsup{a}{}{2}bc\mp@subsup{m}{}{5}\mp@subsup{x}{}{2}+1050A\mp@subsup{a}{}{2}bd\mp@subsup{m}{}{3}\mp@subsup{x}{}{4}+1050Aa\mp@subsup{b}{}{2}c\mp@subsup{m}{}{3}\mp@subsup{x}{}{4}+7731Aa\mp@subsup{b}{}{2}dm\mp@subsup{x}{}{6}+2577A\mp@subsup{b}{}{3}cm\mp@subsup{x}{}{6}+B\mp@subsup{a}{}{3}c\mp@subsup{m}{}{5}\mp@subsup{x}{}{2}
+350B\mp@subsup{a}{}{3}d\mp@subsup{m}{}{3}\mp@subsup{x}{}{4}+1050B\mp@subsup{a}{}{2}bc\mp@subsup{m}{}{3}\mp@subsup{x}{}{4}+7731B\mp@subsup{a}{}{2}bdm\mp@subsup{x}{}{6}+7731Ba\mp@subsup{b}{}{2}cm\mp@subsup{x}{}{6}+33A\mp@subsup{a}{}{3}d\mp@subsup{m}{}{4}\mp@subsup{x}{}{2}+99A\mp@subsup{a}{}{2}bc\mp@subsup{m}{}{4}\mp@subsup{x}{}{2}+5190A\mp@subsup{a}{}{2}bd\mp@subsup{m}{}{2}\mp@subsup{x}{}{4}+5190Aa\mp@subsup{b}{}{2}c\mp@subsup{m}{}{2}\mp@subsup{x}{}{4}
+4455Aa\mp@subsup{b}{}{2}d\mp@subsup{x}{}{6}+1485A\mp@subsup{b}{}{3}c\mp@subsup{x}{}{6}+33B\mp@subsup{a}{}{3}c\mp@subsup{m}{}{4}\mp@subsup{x}{}{2}+1730B\mp@subsup{a}{}{3}d\mp@subsup{m}{}{2}\mp@subsup{x}{}{4}+5190B\mp@subsup{a}{}{2}bc\mp@subsup{m}{}{2}\mp@subsup{x}{}{4}+4455B\mp@subsup{a}{}{2}bd\mp@subsup{x}{}{6}+4455Ba\mp@subsup{b}{}{2}c\mp@subsup{x}{}{6}+A\mp@subsup{a}{}{3}c\mp@subsup{m}{}{5}+406A\mp@subsup{a}{}{3}d\mp@subsup{m}{}{3}\mp@subsup{x}{}{2}
+1218A a b bcm 3}\mp@subsup{x}{}{2}+10467A\mp@subsup{a}{}{2}bdm\mp@subsup{x}{}{4}+10467Aa\mp@subsup{b}{}{2}cm\mp@subsup{x}{}{4}+406B\mp@subsup{a}{}{3}c\mp@subsup{m}{}{3}\mp@subsup{x}{}{2}+3489B\mp@subsup{a}{}{3}dm\mp@subsup{x}{}{4}+10467B\mp@subsup{a}{}{2}bcm\mp@subsup{x}{}{4}+35A\mp@subsup{a}{}{3}c\mp@subsup{m}{}{4}+2262A\mp@subsup{a}{}{3}d\mp@subsup{m}{}{2}\mp@subsup{x}{}{2
+6786A a bcm ' }\mp@subsup{x}{}{2}+6237A\mp@subsup{a}{}{2}bd\mp@subsup{x}{}{4}+6237Aa\mp@subsup{b}{}{2}c\mp@subsup{x}{}{4}+2262B\mp@subsup{a}{}{3}c\mp@subsup{m}{}{2}\mp@subsup{x}{}{2}+2079B\mp@subsup{a}{}{3}d\mp@subsup{x}{}{4}+6237B\mp@subsup{a}{}{2}bc\mp@subsup{x}{}{4}+470A\mp@subsup{a}{}{3}c\mp@subsup{m}{}{3}+5353A\mp@subsup{a}{}{3}dm\mp@subsup{x}{}{2
+16059A\mp@subsup{a}{}{2}bcm\mp@subsup{x}{}{2}+5353B\mp@subsup{a}{}{3}cm\mp@subsup{x}{}{2}+3010A\mp@subsup{a}{}{3}c\mp@subsup{m}{}{2}+3465A\mp@subsup{a}{}{3}d\mp@subsup{x}{}{2}+10395A\mp@subsup{a}{}{2}bc\mp@subsup{x}{}{2}+3465B\mp@subsup{a}{}{3}c\mp@subsup{x}{}{2}+9129A\mp@subsup{a}{}{3}cm+10395Ac\mp@subsup{a}{}{3})(ex\mp@subsup{)}{}{m})
```

Problem 2: Result more than twice size of optimal antiderivative.

$$
\int(e x)^{m}\left(b x^{2}+a\right)\left(B x^{2}+A\right)\left(d x^{2}+c\right) \mathrm{d} x
$$

Optimal(type 3, 97 leaves, 2 steps):

$$
\frac{a A c(e x)^{1+m}}{e(1+m)}+\frac{(a A d+A b c+a B c)(e x)^{3+m}}{e^{3}(3+m)}+\frac{(A b d+a B d+b B c)(e x)^{5+m}}{e^{5}(5+m)}+\frac{b B d(e x)^{7+m}}{e^{7}(7+m)}
$$

Result(type 3, 320 leaves):
$\frac{1}{(7+m)(5+m)(3+m)(1+m)}\left(x\left(B b d m^{3} x^{6}+9 B b d m^{2} x^{6}+A b d m^{3} x^{4}+B a d m^{3} x^{4}+B b c m^{3} x^{4}+23 B b d m x^{6}+11 A b d m^{2} x^{4}+11 B a d m^{2} x^{4}\right.\right.$
$+11 B b c m^{2} x^{4}+15 B b d x^{6}+A a d m^{3} x^{2}+A b c m^{3} x^{2}+31 A b d m x^{4}+B a c m^{3} x^{2}+31 B a d m x^{4}+31 B b c m x^{4}+13 A a d m^{2} x^{2}+13 A b c m^{2} x^{2}$
$+21 A b d x^{4}+13 B a c m^{2} x^{2}+21 B a d x^{4}+21 B b c x^{4}+A a c m^{3}+47 A a d m x^{2}+47 A b c m x^{2}+47 B a c m x^{2}+15 A a c m^{2}+35 A a d x^{2}+35 A b c x^{2}$
$\left.\left.+35 B a c x^{2}+71 A a c m+105 A a c\right)(e x)^{m}\right)$

Problem 3: Unable to integrate problem.

$$
\int \frac{(e x)^{m}\left(B x^{2}+A\right)\left(d x^{2}+c\right)}{\left(b x^{2}+a\right)^{3}} \mathrm{~d} x
$$

Optimal(type 5, 201 leaves, 3 steps):

$$
\begin{aligned}
& -\frac{(A b(a d(1-m)-b c(3-m))-a B(b c(1+m)-a d(3+m)))(e x)^{1+m}}{8 a^{2} b^{2} e\left(b x^{2}+a\right)}+\frac{(A b-a B)(e x)^{1+m}\left(d x^{2}+c\right)}{4 a b e\left(b x^{2}+a\right)^{2}} \\
& \quad+\frac{(A b(1-m)(b c(3-m)+a d(1+m))+a B(1+m)(a d(3+m)+b(-c m+c)))(e x)^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{8 a^{3} b^{2} e(1+m)}
\end{aligned}
$$

Result(type 8, 31 leaves):

$$
\int \frac{(e x)^{m}\left(B x^{2}+A\right)\left(d x^{2}+c\right)}{\left(b x^{2}+a\right)^{3}} \mathrm{~d} x
$$

Problem 4: Result more than twice size of optimal antiderivative.

$$
\int(e x)^{m}\left(b x^{2}+a\right)\left(B x^{2}+A\right)\left(d x^{2}+c\right)^{2} \mathrm{~d} x
$$

Optimal(type 3, 144 leaves, 2 steps):
$\frac{a A c^{2}(e x)^{1+m}}{e(1+m)}+\frac{c(2 a A d+A b c+a B c)(e x)^{3+m}}{e^{3}(3+m)}+\frac{(a d(A d+2 B c)+b c(2 A d+B c))(e x)^{5+m}}{e^{5}(5+m)}+\frac{d(A b d+a B d+2 b B c)(e x)^{7+m}}{e^{7}(7+m)}$

$$
+\frac{b B d^{2}(e x)^{9+m}}{e^{9}(9+m)}
$$

Result(type 3, 710 leaves):
$\frac{1}{(9+m)(7+m)(5+m)(3+m)(1+m)}\left(x\left(B b d^{2} m^{4} x^{8}+16 B b d^{2} m^{3} x^{8}+A b d^{2} m^{4} x^{6}+B a d^{2} m^{4} x^{6}+2 B b c d m^{4} x^{6}+86 B b d^{2} m^{2} x^{8}+18 A b d^{2} m^{3} x^{6}\right.\right.$
$+18 B a d^{2} m^{3} x^{6}+36 B b c d m^{3} x^{6}+176 B b d^{2} m x^{8}+A a d^{2} m^{4} x^{4}+2 A b c d m^{4} x^{4}+104 A b d^{2} m^{2} x^{6}+2 B a c d m^{4} x^{4}+104 B a d^{2} m^{2} x^{6}+B b c^{2} m^{4} x^{4}$
$+208 B b c d m^{2} x^{6}+105 b B d^{2} x^{8}+20 A a d^{2} m^{3} x^{4}+40 A b c d m^{3} x^{4}+222 A b d^{2} m x^{6}+40 B a c d m^{3} x^{4}+222 B a d^{2} m x^{6}+20 B b c^{2} m^{3} x^{4}+444 B b c d m x^{6}$
$+2 A a c d m^{4} x^{2}+130 A a d^{2} m^{2} x^{4}+A b c^{2} m^{4} x^{2}+260 A b c d m^{2} x^{4}+135 A b d^{2} x^{6}+B a c^{2} m^{4} x^{2}+260 B a c d m^{2} x^{4}+135 B a d^{2} x^{6}+130 B b c^{2} m^{2} x^{4}$
$+270 B b c d x^{6}+44 A a c d m^{3} x^{2}+300 A a d^{2} m x^{4}+22 A b c^{2} m^{3} x^{2}+600 A b c d m x^{4}+22 B a c^{2} m^{3} x^{2}+600 B a c d m x^{4}+300 B b c^{2} m x^{4}+A a c^{2} m^{4}$
$+328 A a c d m^{2} x^{2}+189 A a d^{2} x^{4}+164 A b c^{2} m^{2} x^{2}+378 A b c d x^{4}+164 B a c^{2} m^{2} x^{2}+378 B a c d x^{4}+189 B b c^{2} x^{4}+24 A a c^{2} m^{3}+916 A a c d m x^{2}$
$\left.\left.+458 A b c^{2} m x^{2}+458 B a c^{2} m x^{2}+206 A a c^{2} m^{2}+630 A a c d x^{2}+315 A b c^{2} x^{2}+315 B a c^{2} x^{2}+744 A a c^{2} m+945 a A c^{2}\right)(e x)^{m}\right)$

Problem 5: Result more than twice size of optimal antiderivative.

$$
\int(e x)^{m}\left(b x^{2}+a\right)^{3}\left(B x^{2}+A\right)\left(d x^{2}+c\right)^{3} \mathrm{~d} x
$$

Optimal(type 3, 379 leaves, 2 steps):

$$
\begin{aligned}
& \frac{a^{3} A c^{3}(e x)^{1+m}}{e(1+m)}+\frac{a^{2} c^{2}(a B c+3 A(a d+b c))(e x)^{3+m}}{e^{3}(3+m)}+\frac{3 a c\left(a B c(a d+b c)+A\left(a^{2} d^{2}+3 a c b d+b^{2} c^{2}\right)\right)(e x)^{5+m}}{e^{5}(5+m)} \\
& \quad+\frac{\left(3 a B c\left(a^{2} d^{2}+3 a c b d+b^{2} c^{2}\right)+A\left(a^{3} d^{3}+9 a^{2} b c d^{2}+9 a b^{2} c^{2} d+b^{3} c^{3}\right)\right)(e x)^{7+m}}{e^{7}(7+m)} \\
& \quad+\frac{\left(a^{3} B d^{3}+9 a b^{2} c d(A d+B c)+3 a^{2} b d^{2}(A d+3 B c)+b^{3} c^{2}(3 A d+B c)\right)(e x)^{9+m}}{e^{9}(9+m)} \\
& \quad+\frac{3 b d\left(a^{2} B d^{2}+b^{2} c(A d+B c)+a b d(A d+3 B c)\right)(e x)^{11+m}}{e^{11}(11+m)}+\frac{b^{2} d^{2}(A b d+3 a B d+3 b B c)(e x)^{13+m}}{e^{13}(13+m)}+\frac{b^{3} B d^{3}(e x)^{15+m}}{e^{15}(15+m)}
\end{aligned}
$$

Result(type ?, 3952 leaves): Display of huge result suppressed!
Problem 6: Result more than twice size of optimal antiderivative.

$$
\int(e x)^{m}\left(b x^{2}+a\right)^{2}\left(B x^{2}+A\right)\left(d x^{2}+c\right)^{3} \mathrm{~d} x
$$

Optimal(type 3, 284 leaves, 2 steps):

$$
\begin{aligned}
& \frac{a^{2} A c^{3}(e x)^{1+m}}{e(1+m)}+\frac{a c^{2}(3 a A d+2 A b c+a B c)(e x)^{3+m}}{e^{3}(3+m)}+\frac{c\left(a B c(3 a d+2 b c)+A\left(3 a^{2} d^{2}+6 a c b d+b^{2} c^{2}\right)\right)(e x)^{5+m}}{e^{5}(5+m)} \\
& \quad+\frac{\left(6 a b c d(A d+B c)+a^{2} d^{2}(A d+3 B c)+b^{2} c^{2}(3 A d+B c)\right)(e x)^{7+m}}{e^{7}(7+m)}+\frac{d\left(a^{2} B d^{2}+3 b^{2} c(A d+B c)+2 a b d(A d+3 B c)\right)(e x)^{9+m}}{e^{9}(9+m)} \\
& \quad+\frac{b d^{2}(A b d+2 a B d+3 b B c)(e x)^{11+m}}{e^{11}(11+m)}+\frac{b^{2} B d^{3}(e x)^{13+m}}{e^{13}(13+m)}
\end{aligned}
$$

Result(type ?, 2442 leaves): Display of huge result suppressed!
Problem 7: Result more than twice size of optimal antiderivative.

$$
\int(e x)^{m}\left(b x^{2}+a\right)\left(B x^{2}+A\right)\left(d x^{2}+c\right)^{3} \mathrm{~d} x
$$

Optimal(type 3, 189 leaves, 2 steps):

$$
\begin{aligned}
& \frac{a A c^{3}(e x)^{1+m}}{e(1+m)}+\frac{c^{2}(3 a A d+A b c+a B c)(e x)^{3+m}}{e^{3}(3+m)}+\frac{c(3 a d(A d+B c)+b c(3 A d+B c))(e x)^{5+m}}{e^{5}(5+m)} \\
& \quad+\frac{d(3 b c(A d+B c)+a d(A d+3 B c))(e x)^{7+m}}{e^{7}(7+m)}+\frac{d^{2}(A b d+a B d+3 b B c)(e x)^{9+m}}{e^{9}(9+m)}+\frac{b B d^{3}(e x)^{11+m}}{e^{11}(11+m)}
\end{aligned}
$$

Result(type 3, 1228 leaves):

$$
\begin{aligned}
& 1 \\
(11 & +m)(9+m)(7+m)(5+m)(3+m)(1+m) \\
& +27 A b d^{3} m^{4} x^{8}+27 B a d^{3} m^{4} x^{8}+81 B b c d^{2} m^{4} x^{8}+950 B b d^{3} m^{2} x^{10}+A a d^{5} m^{5} x^{6}+3 A b c d^{2} m^{5} x^{6}+262 A b d^{3} m^{3} x^{8}+3 B a c d^{2} m^{5} x^{6} \\
& +262 B a d^{3} m^{3} x^{8}+3 B b c^{2} d m^{5} x^{6}+786 B b c d^{2} m^{3} x^{8}+1689 B b d^{3} m x^{10}+29 A a d^{3} m^{4} x^{6}+87 A b c d^{2} m^{4} x^{6}+1122 A b d^{3} m^{2} x^{8}+87 B a c d^{2} m^{4} x^{6} \\
& +1122 B a d^{3} m^{2} x^{8}+87 B b c^{2} d m^{4} x^{6}+3366 B b c d^{2} m^{2} x^{8}+945 b B d^{3} x^{10}+3 A a c d^{2} m^{5} x^{4}+302 A a d^{3} m^{3} x^{6}+3 A b c^{2} d m^{5} x^{4}+906 A b c d^{2} m^{3} x^{6} \\
& +2041 A b d^{3} m x^{8}+3 B a c^{2} d m^{5} x^{4}+906 B a c d^{2} m^{3} x^{6}+2041 B a d^{3} m x^{8}+B b c^{3} m^{5} x^{4}+906 B b c^{2} d m^{3} x^{6}+6123 B b c d^{2} m x^{8}+93 A a c d^{2} m^{4} x^{4} \\
& +1366 A a d^{3} m^{2} x^{6}+93 A b c^{2} d m^{4} x^{4}+4098 A b c d^{2} m^{2} x^{6}+1155 A b d^{3} x^{8}+93 B a c^{2} d m^{4} x^{4}+4098 B a c d^{2} m^{2} x^{6}+1155 B a d^{3} x^{8}+31 B b c^{3} m^{4} x^{4} \\
& +4098 B b c^{2} d m^{2} x^{6}+3465 B b c d^{2} x^{8}+3 A a c^{2} d m^{5} x^{2}+1050 A a c d^{2} m^{3} x^{4}+2577 A a d^{3} m x^{6}+A b c^{3} m^{5} x^{2}+1050 A b c^{2} d m^{3} x^{4}+7731 A b c d^{2} m x^{6} \\
& +B a c^{3} m^{5} x^{2}+1050 B a c^{2} d m^{3} x^{4}+7731 B a c d^{2} m x^{6}+350 B b c^{3} m^{3} x^{4}+7731 B b c^{2} d m x^{6}+99 A a c^{2} d m^{4} x^{2}+5190 A a c d^{2} m^{2} x^{4}+1485 A a d^{3} x^{6} \\
& +33 A b{c^{3} m^{4} x^{2}+5190 A b c^{2} d m^{2} x^{4}+4455 A b c d^{2} x^{6}+33 B a c^{3} m^{4} x^{2}+5190 B a c^{2} d m^{2} x^{4}+4455 B a c d^{2} x^{6}+1730 B b c^{3} m^{2} x^{4}+4455 B b c^{2} d x^{6}}+A a c^{3} m^{5}+1218 A a c^{2} d m^{3} x^{2}+10467 A a c d^{2} m x^{4}+406 A b c^{3} m^{3} x^{2}+10467 A b c^{2} d m x^{4}+406 B a c^{3} m^{3} x^{2}+10467 B a c^{2} d m x^{4}+3489 B b c^{3} m x^{4} \\
& +35 A a c^{3} m^{4}+6786 A a c^{2} d m^{2} x^{2}+6237 A a c d^{2} x^{4}+2262 A b c^{3} m^{2} x^{2}+6237 A b c^{2} d x^{4}+2262 B a c^{3} m^{2} x^{2}+6237 B a c^{2} d x^{4}+2079 B b c^{3} x^{4} \\
& +470 A a c^{3} m^{3}+16059 A a c^{2} d m x^{2}+5353 A b c^{3} m x^{2}+5353 B a c^{3} m x^{2}+3010 A a c^{3} m^{2}+10395 A a c^{2} d x^{2}+3465 A b c^{3} x^{2}+3465 B a c^{3} x^{2} \\
& \left.\left.+9129 A a c^{3} m+10395 a A c^{3}\right)(e x)^{m}\right)
\end{aligned}
$$

Problem 8: Result more than twice size of optimal antiderivative.

$$
\int(e x)^{m}\left(B x^{2}+A\right)\left(d x^{2}+c\right)^{3} \mathrm{~d} x
$$

Optimal(type 3, 121 leaves, 2 steps):

$$
\frac{A c^{3}(e x)^{1+m}}{e(1+m)}+\frac{c^{2}(3 A d+B c)(e x)^{3+m}}{e^{3}(3+m)}+\frac{3 c d(A d+B c)(e x)^{5+m}}{e^{5}(5+m)}+\frac{d^{2}(A d+3 B c)(e x)^{7+m}}{e^{7}(7+m)}+\frac{B d^{3}(e x)^{9+m}}{e^{9}(9+m)}
$$

Result(type 3, 474 leaves):
$\frac{1}{(9+m)(7+m)(5+m)(3+m)(1+m)}\left(x\left(B d^{3} m^{4} x^{8}+16 B d^{3} m^{3} x^{8}+A d^{3} m^{4} x^{6}+3 B c d^{2} m^{4} x^{6}+86 B d^{3} m^{2} x^{8}+18 A d^{3} m^{3} x^{6}+54 B c d^{2} m^{3} x^{6}\right.\right.$
$+176 B d^{3} m x^{8}+3 A c d^{2} m^{4} x^{4}+104 A d^{3} m^{2} x^{6}+3 B c^{2} d m^{4} x^{4}+312 B c d^{2} m^{2} x^{6}+105 B d^{3} x^{8}+60 A c d^{2} m^{3} x^{4}+222 A d^{3} m x^{6}+60 B c^{2} d m^{3} x^{4}$
$+666 B c d^{2} m x^{6}+3 A c^{2} d m^{4} x^{2}+390 A c d^{2} m^{2} x^{4}+135 A d^{3} x^{6}+B c^{3} m^{4} x^{2}+390 B c^{2} d m^{2} x^{4}+405 B c d^{2} x^{6}+66 A c^{2} d m^{3} x^{2}+900 A c d^{2} m x^{4}$
$+22 B c^{3} m^{3} x^{2}+900 B c^{2} d m x^{4}+A c^{3} m^{4}+492 A c^{2} d m^{2} x^{2}+567 A c d^{2} x^{4}+164 B c^{3} m^{2} x^{2}+567 B c^{2} d x^{4}+24 A c^{3} m^{3}+1374 A c^{2} d m x^{2}+458 B c^{3} m x^{2}$
$\left.\left.+206 A c^{3} m^{2}+945 A c^{2} d x^{2}+315 B c^{3} x^{2}+744 A c^{3} m+945 A c^{3}\right)(e x)^{m}\right)$

Problem 9: Unable to integrate problem.

$$
\int \frac{(e x)^{m}\left(B x^{2}+A\right)\left(d x^{2}+c\right)^{3}}{b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 5, 256 leaves, 3 steps):
$-\frac{\left(a^{3} B d^{3}+3 a b^{2} c d(A d+B c)-a^{2} b d^{2}(A d+3 B c)-b^{3} c^{2}(3 A d+B c)\right)(e x)^{1+m}}{b^{4} e(1+m)}+\frac{d\left(a^{2} B d^{2}+3 b^{2} c(A d+B c)-a b d(A d+3 B c)\right)(e x)^{3+m}}{b^{3} e^{3}(3+m)}$
$+\frac{d^{2}(A b d-a B d+3 b B c)(e x)^{5+m}}{b^{2} e^{5}(5+m)}+\frac{B d^{3}(e x)^{7+m}}{b e^{7}(7+m)}+\frac{(A b-a B)(-a d+b c)^{3}(e x)^{1+m} \text { hypergeom }\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{a b^{4} e(1+m)}$
Result(type 8, 33 leaves):

$$
\int \frac{(e x)^{m}\left(B x^{2}+A\right)\left(d x^{2}+c\right)^{3}}{b x^{2}+a} \mathrm{~d} x
$$

Problem 10: Unable to integrate problem.

$$
\int \frac{(e x)^{m}\left(b x^{2}+a\right)^{3}\left(B x^{2}+A\right)}{d x^{2}+c} \mathrm{~d} x
$$

Optimal(type 5, 258 leaves, 3 steps):
$\frac{\left(a^{3} B d^{3}-b^{3} c^{2}(-A d+B c)+3 a b^{2} c d(-A d+B c)-3 a^{2} b d^{2}(-A d+B c)\right)(e x)^{1+m}}{d^{4} e(1+m)}$
$+\frac{b\left(3 a^{2} B d^{2}+b^{2} c(-A d+B c)-3 a b d(-A d+B c)\right)(e x)^{3+m}}{d^{3} e^{3}(3+m)}-\frac{b^{2}(-A b d-3 a B d+b B c)(e x)^{5+m}}{d^{2} e^{5}(5+m)}+\frac{b^{3} B(e x)^{7+m}}{d e^{7}(7+m)}$
$+\frac{(-a d+b c)^{3}(-A d+B c)(e x)^{1+m} \text { hypergeom }\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{d x^{2}}{c}\right)}{c d^{4} e(1+m)}$
Result(type 8, 33 leaves):

$$
\int \frac{(e x)^{m}\left(b x^{2}+a\right)^{3}\left(B x^{2}+A\right)}{d x^{2}+c} \mathrm{~d} x
$$

Problem 11: Unable to integrate problem.

$$
\int \frac{(e x)^{m}\left(B x^{2}+A\right)}{\left(b x^{2}+a\right)^{3}\left(d x^{2}+c\right)} \mathrm{d} x
$$

Optimal(type 5, 332 leaves, 6 steps):

$$
\begin{aligned}
& \frac{(A b-a B)(e x)^{1+m}}{4 a(-a d+b c) e\left(b x^{2}+a\right)^{2}}+\frac{(A b(b c(3-m)-a d(7-m))+a B(a d(3-m)+b c(1+m)))(e x)^{1+m}}{8 a^{2}(-a d+b c)^{2} e\left(b x^{2}+a\right)} \\
& +\frac{1}{8 a^{3}(-a d+b c)^{3} e(1+m)}\left(\left(A b\left(a^{2} d^{2}\left(m^{2}-8 m+15\right)-2 a b c d\left(m^{2}-6 m+5\right)+b^{2} c^{2}\left(m^{2}-4 m+3\right)\right)+a B\left(b^{2} c^{2}\left(-m^{2}+1\right)-2 a b c d( \right.\right.\right. \\
& \left.\left.\left.\left.-m^{2}+2 m+3\right)-a^{2} d^{2}\left(m^{2}-4 m+3\right)\right)\right)(e x)^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)\right) \\
& +\frac{d^{2}(-A d+B c)(e x)^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{d x^{2}}{c}\right)}{c(-a d+b c)^{3} e(1+m)}
\end{aligned}
$$

Result (type 8, 33 leaves):

$$
\int \frac{(e x)^{m}\left(B x^{2}+A\right)}{\left(b x^{2}+a\right)^{3}\left(d x^{2}+c\right)} \mathrm{d} x
$$

Problem 12: Unable to integrate problem.

$$
\int \frac{(e x)^{m}\left(B x^{2}+A\right)}{\left(b x^{2}+a\right)^{2}\left(d x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Optimal(type 5, 292 leaves, 6 steps):

$$
\begin{aligned}
& \frac{d(a A d+A b c-2 a B c)(e x)^{1+m}}{2 a c(-a d+b c)^{2} e\left(d x^{2}+c\right)}+\frac{(A b-a B)(e x)^{1+m}}{2 a(-a d+b c) e\left(b x^{2}+a\right)\left(d x^{2}+c\right)} \\
& \quad+\frac{b(A b(b c(1-m)-a d(5-m))+a B(a d(3-m)+b c(1+m)))(e x)^{1+m} \text { hypergeom }\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{2 a^{2}(-a d+b c)^{3} e(1+m)} \\
& \quad-\frac{d(b c(B c(3-m)-A d(5-m))+a d(A d(1-m)+B c(1+m)))(e x)^{1+m} \text { hypergeom }\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{d x^{2}}{c}\right)}{2 c^{2}(-a d+b c)^{3} e(1+m)}
\end{aligned}
$$

Result(type 8, 33 leaves):

$$
\int \frac{(e x)^{m}\left(B x^{2}+A\right)}{\left(b x^{2}+a\right)^{2}\left(d x^{2}+c\right)^{2}} \mathrm{~d} x
$$

Problem 13: Unable to integrate problem.

$$
\int \frac{(e x)^{m}\left(b x^{2}+a\right)\left(B x^{2}+A\right)}{\left(d x^{2}+c\right)^{3}} \mathrm{~d} x
$$

Optimal(type 5, 200 leaves, 3 steps):

$$
\begin{aligned}
& -\frac{(-a d+b c)(e x)^{1+m}\left(B x^{2}+A\right)}{4 c d e\left(d x^{2}+c\right)^{2}}+\frac{(b c(A d(1+m)-B c(3+m))+a d(A d(3-m)-B(-c m+c)))(e x)^{1+m}}{8 c^{2} d^{2} e\left(d x^{2}+c\right)} \\
& \quad+\frac{(a d(1-m)(A d(3-m)+B c(1+m))+b c(1+m)(A d(1-m)+B c(3+m)))(e x)^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{d x^{2}}{c}\right)}{8 c^{3} d^{2} e(1+m)}
\end{aligned}
$$

Result(type 8, 31 leaves):

$$
\int \frac{(e x)^{m}\left(b x^{2}+a\right)\left(B x^{2}+A\right)}{\left(d x^{2}+c\right)^{3}} \mathrm{~d} x
$$

Problem 14: Unable to integrate problem.

$$
\int \frac{(e x)^{m}\left(B x^{2}+A\right)}{\left(b x^{2}+a\right)\left(d x^{2}+c\right)^{3}} d x
$$

Optimal(type 5, 323 leaves, 6 steps):

$$
\frac{(-A d+B c)(e x)^{1+m}}{4 c(-a d+b c) e\left(d x^{2}+c\right)^{2}}+\frac{(b c(B c(3-m)-A d(7-m))+a d(A d(3-m)+B c(1+m)))(e x)^{1+m}}{8 c^{2}(-a d+b c)^{2} e\left(d x^{2}+c\right)}
$$

$$
+\frac{b^{2}(A b-a B)(e x)^{1+m} \text { hypergeom }\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{a(-a d+b c)^{3} e(1+m)}+\frac{1}{8 c^{3}(-a d+b c)^{3} e(1+m)}\left(\left(b^{2} c^{2}(B c(1-m)-A d(5\right.\right.
$$

$$
\left.-m))(3-m)-a^{2} d^{2}(1-m)(A d(3-m)+B c(1+m))+2 a b c d\left(B c\left(-m^{2}+2 m+3\right)+A d\left(m^{2}-6 m+5\right)\right)\right)(e x)^{1+m} \text { hypergeom }\left(\left[1, \frac{1}{2}\right.\right.
$$

$$
\left.\left.\left.+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{d x^{2}}{c}\right)\right)
$$

Result(type 8, 33 leaves):

$$
\int \frac{(e x)^{m}\left(B x^{2}+A\right)}{\left(b x^{2}+a\right)\left(d x^{2}+c\right)^{3}} d x
$$

Problem 15: Unable to integrate problem.

$$
\int(e x)^{m}\left(b x^{2}+a\right)^{p}\left(B x^{2}+A\right)\left(d x^{2}+c\right)^{2} \mathrm{~d} x
$$

Optimal(type 5, 493 leaves, 5 steps):

$$
\begin{aligned}
& \frac{1}{b^{3} e(3+m+2 p)(5+m+2 p)(7+m+2 p)}\left(\left(a^{2} B d^{2}\left(m^{2}+8 m+15\right)+b^{2} c\left(8 B c+A d(7+m+2 p)^{2}\right)-a b d(A d(3+m)(7+m+2 p)\right.\right. \\
& \left.\left.\left.+B c\left(27+m^{2}+2 p+2 m(6+p)\right)\right)\right)(e x)^{1+m}\left(b x^{2}+a\right)^{1+p}\right) \\
& -\frac{(a B d(5+m)-b(4 B c+A d(7+m+2 p)))(e x)^{1+m}\left(b x^{2}+a\right)^{1+p}\left(d x^{2}+c\right)}{b^{2} e(5+m+2 p)(7+m+2 p)}+\frac{B(e x)^{1+m}\left(b x^{2}+a\right)^{1+p}\left(d x^{2}+c\right)^{2}}{b e(7+m+2 p)} \\
& -\frac{1}{b^{3} e(1+m)(3+m+2 p)(5+m+2 p)(7+m+2 p)\left(1+\frac{b x^{2}}{a}\right)^{p}}((b c(3+m+2 p)(2 b c(2+p)(a B(1+m)-A b(7+m+2 p))+(
\end{aligned}
$$

$$
-a d+b c)(1+m)(a B(5+m)-A b(7+m+2 p)))-a(1+m)(2 b c d(2+p)(a B(1+m)-A b(7+m+2 p))+d(-a d+b c)(1
$$

$$
+m)(a B(5+m)-A b(7+m+2 p))+2(-a d+b c)(a B d(5+m)-b(4 B c+A d(7+m+2 p)))))(e x)^{1+m}\left(b x^{2}+a\right)^{p} \text { hypergeom }\left(\left[-p, \frac{1}{2}\right.\right.
$$

$$
\left.\left.\left.+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)\right)
$$

Result(type 8, 33 leaves):

$$
\int(e x)^{m}\left(b x^{2}+a\right)^{p}\left(B x^{2}+A\right)\left(d x^{2}+c\right)^{2} \mathrm{~d} x
$$

Problem 16: Unable to integrate problem.

$$
\int \frac{(e x)^{m}\left(b x^{2}+a\right)^{p}\left(B x^{2}+A\right)}{d x^{2}+c} \mathrm{~d} x
$$

Optimal(type 6, 156 leaves, 6 steps):

$$
-\frac{(-A d+B c)(e x)^{1+m}\left(b x^{2}+a\right)^{p} \text { AppellF1 }\left(\frac{1}{2}+\frac{m}{2},-p, 1, \frac{3}{2}+\frac{m}{2},-\frac{b x^{2}}{a},-\frac{d x^{2}}{c}\right)}{c d e(1+m)\left(1+\frac{b x^{2}}{a}\right)^{p}}
$$

Result(type 8, 33 leaves):

$$
\int \frac{(e x)^{m}\left(b x^{2}+a\right)^{p}\left(B x^{2}+A\right)}{d x^{2}+c} \mathrm{~d} x
$$

Problem 17: Unable to integrate problem.

$$
\int \frac{(e x)^{m}\left(b x^{2}+a\right)^{p}\left(B x^{2}+A\right)}{\left(d x^{2}+c\right)^{3}} \mathrm{~d} x
$$

Optimal(type 6, 469 leaves, 8 steps):

$$
\frac{(-A d+B c)(e x)^{1+m}\left(b x^{2}+a\right)^{1+p}}{4 c(-a d+b c) e\left(d x^{2}+c\right)^{2}}+\frac{(a d(A d(3-m)+B c(1+m))+b c(B c(1-m-2 p)-A d(5-m-2 p)))(e x)^{1+m}\left(b x^{2}+a\right)^{1+p}}{8 c^{2}(-a d+b c)^{2} e\left(d x^{2}+c\right)}
$$

$$
+\frac{1}{8 c^{3} d(-a d+b c)^{2} e(1+m)\left(1+\frac{b x^{2}}{a}\right)^{p}}\left(\left(a^{2} d^{2}(1-m)(A d(3-m)+B c(1+m))-2 a b c d(B c(1+m)(1-m-2 p)+A d(1-m)(3\right.\right.
$$

$$
\left.-m-2 p))+b^{2} c^{2}(1-m-2 p)(A d(3-m-2 p)+B c(1+m+2 p))\right)(e x)^{1+m}\left(b x^{2}+a\right)^{p} \text { AppellF1 }\left(\frac{1}{2}+\frac{m}{2},-p, 1, \frac{3}{2}+\frac{m}{2},-\frac{b x^{2}}{a}\right.
$$

$$
\left.\left.-\frac{d x^{2}}{c}\right)\right)-\frac{1}{8 c^{2} d(-a d+b c)^{2} e(1+m)\left(1+\frac{b x^{2}}{a}\right)^{p}}(b(a d(A d(3-m)+B c(1+m))+b c(B c(1-m-2 p)-A d(5-m-2 p)))(1
$$

$$
\left.+m+2 p)(e x)^{1+m}\left(b x^{2}+a\right)^{p} \text { hypergeom }\left(\left[-p, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)\right)
$$

Result(type 8, 33 leaves):

$$
\int \frac{(e x)^{m}\left(b x^{2}+a\right)^{p}\left(B x^{2}+A\right)}{\left(d x^{2}+c\right)^{3}} \mathrm{~d} x
$$

Test results for the 48 problems in "1.1.2.8 $P(x)(c x)^{\wedge} m\left(a+b x^{\wedge} 2\right)^{\wedge} p . t x t "$
Problem 19: Unable to integrate problem.

$$
\int \frac{(c x)^{m}(B x+A)}{b x^{2}+a} \mathrm{~d} x
$$

Optimal(type 5, 87 leaves, 3 steps):

$$
\frac{A(c x)^{1+m} \text { hypergeom }\left(\left[1, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-\frac{b x^{2}}{a}\right)}{a c(1+m)}+\frac{B(c x)^{2+m} \text { hypergeom }\left(\left[1,1+\frac{m}{2}\right],\left[\frac{m}{2}+2\right],-\frac{b x^{2}}{a}\right)}{a c^{2}(2+m)}
$$

Result(type 8, 22 leaves):

$$
\int \frac{(c x)^{m}(B x+A)}{b x^{2}+a} \mathrm{~d} x
$$

## Summary of Integration Test Results

770 integration problems


A - 479 optimal antiderivatives
B - 113 more than twice size of optimal antiderivatives
C - 9 unnecessarily complex antiderivatives
D - 169 unable to integrate problems
E - O integration timeouts


[^0]:    Problem 29: Result more than twice size of optimal antiderivative.

[^1]:    Problem 123: Result more than twice size of optimal antiderivative.

[^2]:    Problem 170: Result more than twice size of optimal antiderivative.

